The Role of Auditing in Mediating Firm Conflict*

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Abstract

Auditing traditionally has served investors in certifying the quality of financial reports. We propose an alternative role of auditing: mediating conflicts between competitive firms in a market. We model a contest in which one firm produces output and the other seeks to expropriate that output. Asymmetric information on the producer's type leads to costly conflict in equilibrium, which is inefficient. Firms cannot avoid these fights through side agreements to share output. Our main result shows that an independent third party auditor can eliminate fighting in equilibrium and its associated dead weight losses. This suggests auditing not only informs investors but also alters the strategic behavior of firms. We speculate on some empirical implications of this alternative view.

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1 Introduction

The dominant view of auditing - embraced by standard-setters, investors, and firms - is that it facilitates capital allocation. Investors, the argument goes, need audited financial statements in order to assess the merits of differential investments across a spectrum of firms. Thus, auditing exists primarily in a capital market context. We propose an alternative view: auditing helps mediate conflicts between self-interested parties. Firms compete with one another for resources and customers, and in this perpetual struggle, auditing has the central ability to certify firm quality. Information on quality is valuable not only to outside investors but to other firms in their decisions on whether to engage in costly fights, which take the form of asset expropriation, intellectual property theft, or any other hostile economic action. Auditing, in our view, is a crucial element to mediating these inevitable conflicts.[1]

We propose a model of conflict between two elemental firms, one that produces output (a producer) and another that steals it (a stealer). We operate in a setting absent capital market investment to isolate the effects from conflict alone. On top of this, we layer asymmetric information on the producer who can either be tough or weak. The stealer forms beliefs about the producer's type, and those beliefs will determine whether the stealer fights (expropriates) or leaves the producer alone. We operate in a world without commitment or formal contracts. Rather, the parties must resolve their conflicts through costly contests, such as modern-day litigation, or refuse to fight and leave the other party with the spoils of production. To fix ideas, we first establish the complete information benchmark where the type of the producer is common knowledge to all parties. The unique equilibrium of this game involves no fighting since the stealer will avoid a strong producer but contest a weak producer. The weak producer avoids the fight and concedes the output to the stealer.

Things change under incomplete information. When the stealer's signal is noisy, he does not know whether the producer is strong or weak and instead forms posterior beliefs based on his signal and his prior belief. Though his signal is better than fair, it is still possibly wrong. Under such conditions, we show that there is non-trivial fighting in the

[1] As a motivating example, consider the contest between Apple and Samsung in the high-end mobile device marketplace. Apple first produced the touchscreen iPhone and Samsung subsequently developed the similar Galaxy phones. While the case remains in appeal, the early judgment has rendered Apple $1.05 billion in damages from Samsung in support of Apple's claim that Samsung stole the core touchscreen technology (Apple [2014]).

unique Perfect Bayesian equilibrium. This occurs when the stealer thinks the producer is weak (when he is actually strong) and initiates a contest, and the producer fights back (since he is actually strong).

These contests are dead weight losses on society and fundamentally arise because of the asymmetric information. The Coase Theorem argues that absent transaction costs, individual actors can bargain to efficient outcomes. As such, we propose a natural bargaining game in an attempt to avoid conflict: the producer concedes a share of output upfront to the stealer in order to satisfy his desire for expropriation and potentially forestall conflict. Yet this sharing game unravels: there is no separating equilibrium, and rather an (almost) unique pooling equilibrium in which both types of the producer offer a share of zero. Fundamentally, the asymmetric information blocks the two parties from obtaining the best joint outcome, no conflict.

Finally, we show that certification of the producer's type does in fact bring the game back to efficiency. If there exists a third party that has the ability to certify quality to the market, then the producer will pay a non-zero amount in order to obtain this certification and thereby avoid conflict. Provided this fee is sufficiently small, certification will prevent the costly conflict that drains total surplus. Auditing is the prime example of this certification, as it provides information to both parties that alters their behavior. We conclude with an exploration of the empirical consequences of such a view.

The existing analytical literature on auditing is a constellation of topics and methods. Some papers focus on pricing of audit services (Magee and Tseng [1990]; Morgan and Stocken [1998]), some on the industrial organization of the audit market (Gerakos and Syverson [2015]), and many on the differing effects of liability and damage apportionment rules (Chan and Pae [1998]; Dye [1993]; Deng et al. [2012]; Liu and Wang [2006]; Melumad and Thoman [1990]; Pae and Yoo [2001]; Patterson and Wright [2003].) Some employ capital market models (Kanodia and Mukherji [1994]) and others principal agent models (Hillegeist [1999]; Laux and Newman [2010]; Narayanan [1994]; Thoman [1996]). Some papers explore the independence of auditors (Magee and Tseng [1990]), their conservatism, or their risk decisions (Newman et al. [2001]). However, what nearly all these papers have in common is their presumption that auditing serves to inform investors alone, and none of the papers explore how auditing effects firm behavior in the marketplace.

Much of the auditing theory literature works in an adverse selection setting in which a firm can be multiple types (e.g. Schwartz [1997], Chan and Pae [1998], and Radhakr-

ishnan [1999]). Usually the type of the firm in these papers reflects assets (like Hillegeist [1999]), cash flows (like Narayanan [1994]), or the value to the investor (like Radhakrishnan [1999]). Here, the type of the producer only signals its fitness in a contest with the stealer. This reflects the producer's fundamental tendency towards litigation and aggression, which may be independent of the firm's actual cash flows. The focus here is not on returns for the investor but rather the firm's own ability to handle external economic threats.

The paper that most clearly articulates the dominant view, and stands in stark contrast to ours, is Newman et al. [2005]. Like our paper, it seeks to provide a fundamental explanation of what auditors do, rather than comparing the detailed institutional features of the audit market and liability regimes, like most of the existing literature. Newman et al. [2005] build on the assumptions of the Law and Finance literature (La Porta et al. [1997]; La Porta et al. [1998]; and Porta et al. [2000]) in which self-interested insiders tend to expropriate assets of external shareholders, and auditing is a means of mitigating these agency conflicts. This kind of expropriation occurs between inside managers and external shareholders. We propose expropriation of a different kind: between firms operating in the same market. Some firms are pioneers in innovation and others are copycats. Auditing has a role in deterring this kind of <u>external</u> expropriation (as opposed to the <u>internal</u> expropriation of managers from shareholders).

2 Model

The game in this study is comprised of a producer who procures a resource and possibly confronts a stealer who wishes to expropriate this resource.[2] The stealer need not produce output himself.[3] He can, in fact, serve the function of a non-practicing entity (NPE), a euphemism for the more pejorative "patent trolls": companies that do not produce output themselves but simply acquire technology, license it to others, and sue the original companies from whom they obtained the technology in the first place. Were

[2]The hunter-stealer problem is clearly an evolutionary one, and is widespread among a wide range of species, including current-day primates (Sapolsky [1998, p.358]). This problem is thus likely to have occurred repeatedly in ancient hominid lineage, and has survived in various forms in modern-day societies: the hunter/farmer/enterpreneur/producer has always had to contend with a stealer/marauder/raider/expropriater (e.g., Shleifer et al. [1998]).

[3]The basic unit of analysis is the manager/entrepreneur of each firm, so we use pronouns to describe their behavior.

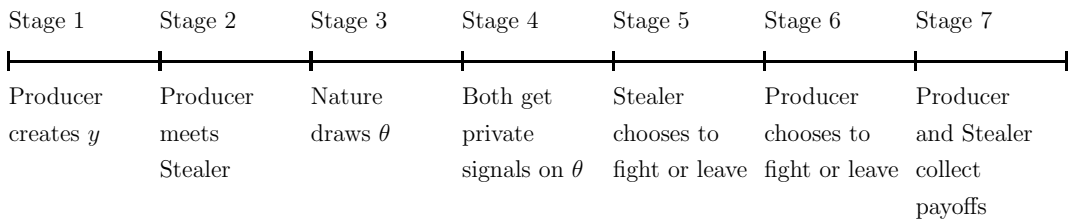

Figure 1: Timeline.

the stealer also to produce output, he would face a similar game when another stealer would threaten to expropriate his assets.

The producer expends effort $e \in [\underline{e}, \bar{e}]$. The production outcome is a stochastic variable in an exogenous range $\tilde{y} \in [\underline{y}, \bar{y}]$ $(0 < \underline{y} < \bar{y})$. We assume that if the producer exerts minimum effort, all sizes of output are equally likely. As the producer starts exerting more effort, larger output becomes increasingly more likely compared to smaller output. We represent this phenomenon with a linear probability density function that is flat at the minimum effort and becomes steeper as the effort level e increases:

$$f(y;e) = \frac{1}{\bar{y} - \underline{y}} \left[(1 - \frac{e - \underline{e}}{\bar{e} - \underline{e}}) + 2 \left(\frac{e - \underline{e}}{\bar{e} - \underline{e}} \right) \left(\frac{y - \underline{y}}{\bar{y} - \underline{y}} \right) \right]. \tag{1}$$

The expected output for an effort level $e \in [\underline{e}, \bar{e}]$ is therefore:

$$\int_{\underline{y}}^{\bar{y}} y f(y;e)\, dy. \tag{2}$$

It can be verified that $f(y;e)$ satisfies the monotone likelihood ratio property (MLRP), i.e., $\frac{\partial}{\partial y} \left(\frac{f_e(y;e)}{f(y,e)} \right) > 0$, which in turn implies first-order stochastic dominance. Consequently, as the producer's effort e increases, the likelihood of higher values of y increases. The effort e costs the producer $C(e)$, where C is a strictly increasing, positive, convex function with $C'(\underline{e}) = 0$ and $C'(\bar{e}) = \infty$.

If the producer gets to consume the output y, the game is over. However, we assume that after acquiring the resource and before consuming it, there is a probability $m \in [0, 1]$ that the producer meets a stealer. The producer and stealer are strangers, and each only cares about his or him personal gain (Nowak and Sigmund [2005, p.1291]). Each must decide whether to fight for the output. Should such a fight occur, the outcome depends

on the relative strengths of the two parties. We model this situation as the producer having a type $\theta \in \Theta = \{\text{Tough, Weak}\}$. The Tough producer will win the fight, and the Weak producer will lose the fight. The type θ is independent of the output level y even though in practice they may be correlated.[4] The types in our model are specific to the contest between the two parties. It is entirely possible that a single firm could be a producer in one context and a stealer in another. For example, in the Apple-Samsung analogy, Samsung may act as a stealer with respect to Apple's iPhone technology but a producer with respect to curved television technology.

Ex-ante, the probability that the producer is Tough is $\phi \in (0, 1)$, and the probability that the producer is Weak is $1 - \phi$. This prior ϕ is common knowledge. When the meeting actually occurs, both parties get private information about the producer's real type. The producer himself gets a perfect private signal of θ, him type, but the stealer gets an imperfect private signal, e.g., based on the producer's outwardly visible state (Parker [1974], Smith and Parker [1976]). The stealer's signal does not come from a third party, like an auditor, but rather is a function of his own information system. For example, the stealer may have some noisy information on the producer's type from various publicly available information sources (like the press) or from word of mouth and other informal channels (we reserve the role of the auditor for later in the paper). This signal σ_s is better than pure noise, i.e., is correct with probability $q_s \in (\frac{1}{2}, 1]$. The stealer uses this signal to update his information on θ from the baseline probability ϕ. Thus,

$$q_s = Prob(\sigma_s = \text{Tough}|\theta = \text{Tough}) = Prob(\sigma_s = \text{Weak}|\theta = \text{Weak}) > \frac{1}{2}. \tag{3}$$

The costs and the benefits to the fight are common knowledge parameters and are as follows. If the producer is Tough (T), he keeps the output but suffers a defense cost μ_{pT}. If the producer is Weak (W), he loses the output and also suffers an injury μ_{pW}. Likewise, the stealer's cost of attacking is μ_{sW} if the producer is Weak and μ_{sT} if the producer is Tough. We assume the Tough producer finds it worthwhile to defend the output; otherwise the game always ends with the producer walking away when confronted, a situation more straightforwardly represented by $\phi = 0$. We therefore specify that the range of $\tilde{y}$ in the game exceeds μ_{pT}, a feature we embed in the model by letting $\underline{y} > \mu_{pT}$.

[4] Ultimately, a producer's ability to fight may arise from the size of the output (since litigation fees must come from somewhere). At the same time, we believe there is a fundamental quality of firms that determines their level of aggression/pugnacity in the economic landscape.

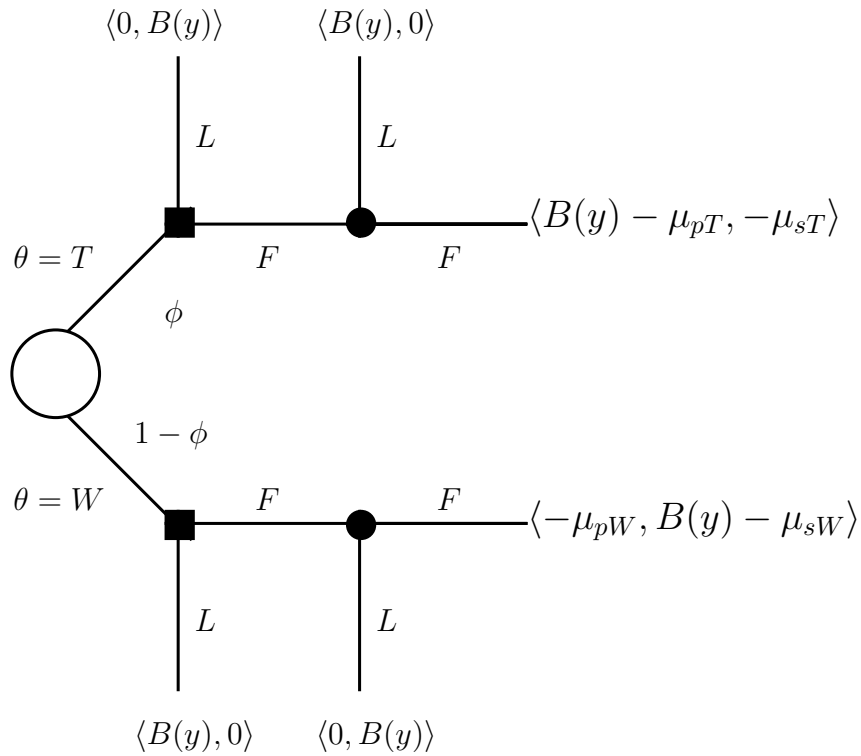

Figure 2: Complete Information Game. The hollow circles represent moves by Nature, the full circles represent moves by the producer, and the squares represent moves by the stealer. The bracketed terms represent the payoffs of the agents. The first term is the producer's payoff, and the second is the stealer's.

The timeline of the fight is in Figure 1.

2.1 Complete Information Benchmark

To fix ideas, consider the benchmark of complete information. Now the information asymmetry falls away, and the equilibrium concept is Subgame Perfect Nash Equilibrium. As necessary in complete information extensive form games, the solution involves backward induction, solving the game from the outermost node and working backwards to the unique equilibrium. When the stealer's signal is accurate ($q_s = 1$), there is no conflict because a stealer will not engage a known Tough producer, and a known Weak producer will always surrender to the stealer. That is:

Proposition 1 *The unique Subgame Perfect Nash Equilibrium of the complete information game involves no fighting in equilibrium:*

1. *When the producer is Tough, the stealer will leave. The producer earns y and the stealer gets nothing.*

2. *When the producer is Weak, the producer will leave. The stealer earns y and the producer gets nothing.*

(All proofs are in the Appendix). The absence of fights implies that that the full-information equilibrium has a social surplus. If all parties can trust each other, they can potentially create a full-information environment and then share the surplus in a manner that each party is better off (in an ex ante sense).

Figure 2 shows the game under perfect information. Because there are no (non-trivial) signals, there is no asymmetric information, and therefore, the equilibrium concept is Subgame Perfect Nash equilibrium. The benchmark here establishes not only the unique equilibrium but also the efficiency of the outcome. This efficiency will erode under asymmetric information.

2.2 The Incomplete Information Game

Now consider the asymmetric information game. The signals now are non-trivial ($q_s < 1$). We now solve the equilibrium by first computing the Perfect Bayesian equilibrium in the stealing game and then backward induct to find the equilibrium effort level in the production game. Figure 3 illustrates the game and the information sets.

Proposition 2 *If the producer with output y and a stealer meet, the unique equilibrium of the stealing game in Figure 3 is:*

1. *If the producer is Weak, he will leave if the stealer attacks. The stealer keeps the output and incurs no fight costs.*

2. *If the producer is Tough, he will fight if the stealer attacks. The producer gets $y - \mu_{pT}$, and the stealer gets $-\mu_{sT}$.*

3. *If the stealer gets the signal Weak, he will attack if and only if:*

$$y > \frac{(1-q_s)\phi}{q_s(1-\phi)}\mu_{sT} \equiv y_W. \tag{4}$$

4. If the stealer gets the signal Tough, he will attack if and only if:

$$y > \frac{q_s \phi}{(1-q_s)(1-\phi)} \mu_{sT} \equiv y_T. \tag{5}$$

Our finding in Proposition 2 was that only some engagements lead to a fight is consistent with prior literature on strategic behavior in survival games (e.g., Smith and Price [1973]). Intuitively, the stealer will attack if he believes the producer is Weak, and equations (4) and (5) capture his posterior beliefs regarding that possibility. In particular, because $q_s > \frac{1}{2}$, $y_T > y_W$, i.e., if the stealer's optimal choice is to attack when he receives the Tough signal, he will also attack when he receives the Weak signal.

The fundamental intuition behind this result is the stealer's distorted inference from his noisy signal. Were his signal accurate, we know from the benchmark case that he would know the producer's type perfectly and, therefore, never attack a strong producer and always attack a weak producer. But when his signal is noisy, there is a non-trivial chance that his signal is wrong: that the producer is strong when the stealer thinks he is weak, or the producer is weak when the stealer thinks he is strong. Under both cases, the stealer will take action that differs from what he would have done under perfect information. The first case leads to costly fights. It is thus the asymmetric information and the concomitant distorted signal that leads to conflict.

The results of the stealing game in Proposition 2 imply that producer will not always get to consume the original output y from the production game. Instead the producer with an original output y gets to consume $B(y)$ where:

$$B(y) = \begin{cases} y & : y \leq y_W \\ m[\phi(1-q_s)(y-\mu_{pT}) + \phi q_s y + (1-\phi)(1-q_s)y] + (1-m)y & : y_W < y \leq y_T \\ m\phi(y-\mu_{pT}) + (1-m)y & : y > y_T. \end{cases} \tag{6}$$

For example, when $y > y_T$ and a stealer appears (which happens with probability m), the stealer will always fight, leaving the producer with $(y - \mu_{pT})$ when the producer is Tough (which happens with probability ϕ) and zero otherwise. Note that $B(y)$ has discontinuous drops at the thresholds y_W and y_T. The reason is that when the stealer elects to fight, the Tough producer incurs a discrete cost of μ_{pT}, and the stealer's decision to fight as a function of his signal changes exactly at these thresholds. In the regions

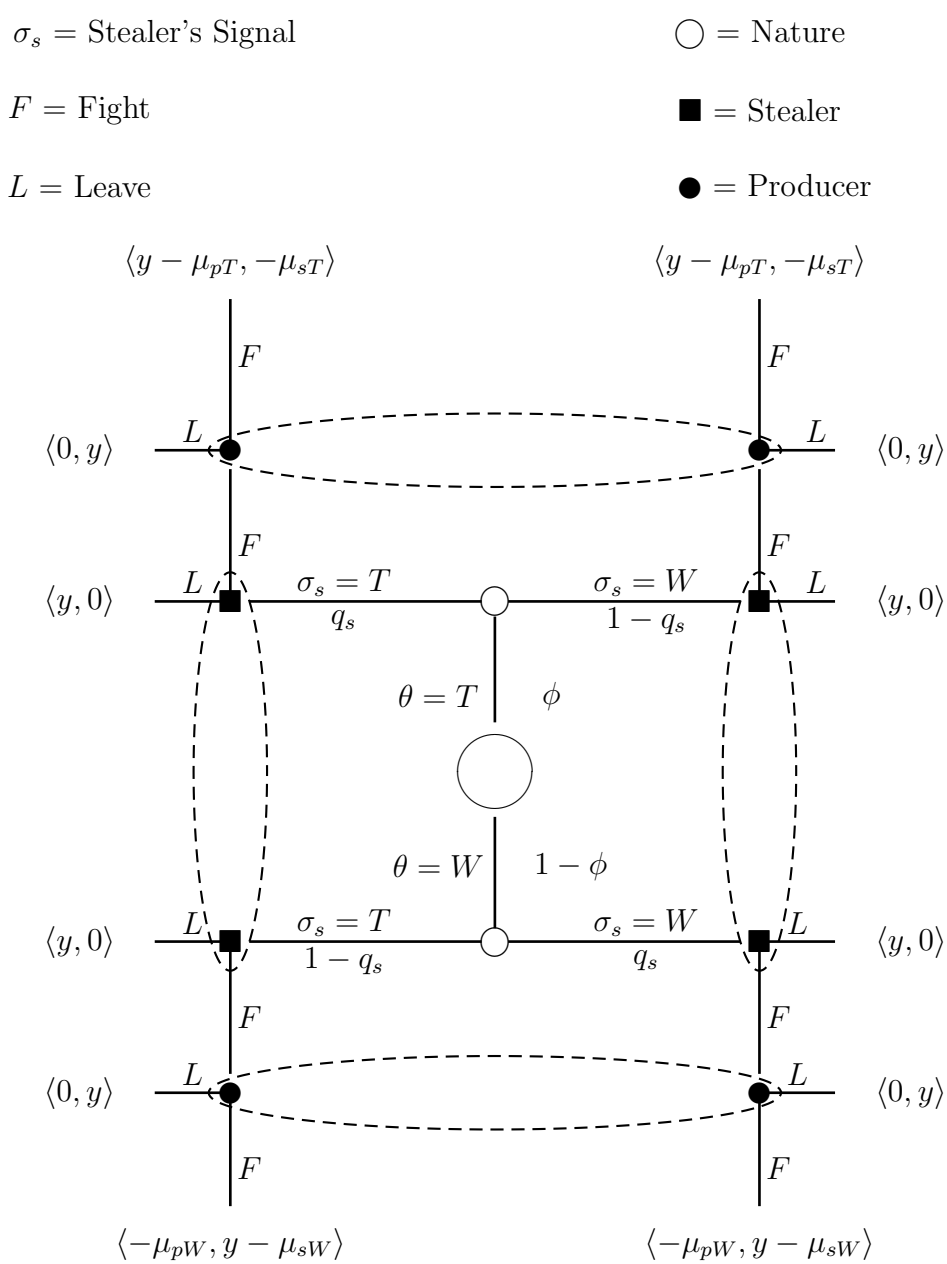

Figure 3: Extensive Form Game. The dotted ovals represent the information sets of each agent.

where it is continuous, $B(y)$ is a linear increasing function in y, indicating that a larger output yields larger gross benefits to the producer.

At the beginning of the game, the producer maximizes:

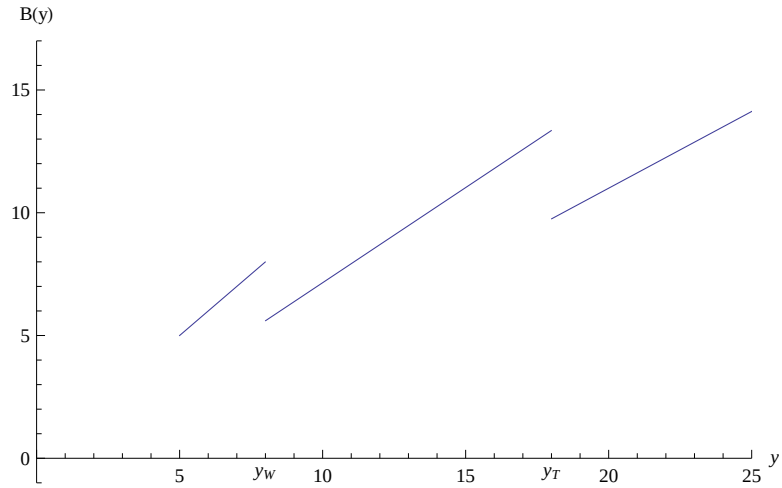

Figure 4: The benefit function $B(y)$ for $m = 0.75$, $\phi = 0.5$, $q_s = 0.6$, $\mu_{pT} = 4$, and $\mu_{sT} = 12$, $y \in [5, 25]$.

$$\max_{e \in [\underline{e}, \bar{e}]} \int_{\underline{y}}^{\bar{y}} B(y) f(y; e)\, dy - C(e). \tag{7}$$

This maximization yields:

Corollary 1 *The production maximization problem in (7) is strictly concave in effort and, therefore, has a unique solution.*

This completes the equilibrium of the game in Figure 3. Note in particular that concavity is not just the concavity of $-C(e)$; it is the concavity of the production function less the cost of effort e.

As a numerical example, consider $m = 0.75$, $\phi = 0.5$, $q_s = 0.6$, $\mu_{pT} = 4$, $\mu_{sT} = 12$, and $y \in [5, 25]$. Figure 4 provides a plot of $B(y)$ for this set of parameters. Note that the discontinuities occur at $y_W = 8$ and $y_T = 18$. In addition, if the cost function $C(e) = -e - ln(1 - e), e \in [0, 1]$, the optimal $e = 0.5768 > 0$.

Modern evolutionary biology and psychology research argues that our preferences for financial and other modern-life resource gambles have been selected through evolutionary resource procurement games (e.g., Cosmides and Tooby [1997]; Gintis [2009]; Trivers [1971]). This perspective finds special salience in Kahneman [2011] who develops the notion of an ancient System 1 in the human brain that was selected for evolutionary

survival but now plays a crucial role in how we approach modern-day gambles over resources. This study therefore builds a non-cooperative evolutionary survival game.

The evolutionary interpretation of our model is that organisms who innately display the behavior in Proposition 2 are more likely to survive in the long run. To do so, this behavior must survive mutations. An evolutionary stable strategy (ESS) is one that is robust to the introduction of mutant strategies (Smith and Price [1973]; Gintis [2009, Ch.10]). We follow this literature by modeling a plausible evolutionary resource procurement and survival game. We use evolutionary game theory where players from some population play a survival game, typically a resource-competition game. The equilibrium strategies in these games (where each party attempts to maximize its share of the resource) are thought of as successful traits or preferences and off-equilibrium strategies as unsuccessful traits or preferences. It is in this manner that preferences are endogenized in this literature (Gintis [2009]; Trivers [1971]).[5]

Corollary 2 *The equilibrium in Proposition 2 is an evolutionarily stable strategy (ESS).*

The ESS equilibrium concept emerges from games that are played repeatedly, where populations are drawn from random and participants fight against each other over time, with the optimal strategy emerging from an infinite number of contests. While repeated play is not an explicit part of our game here, we include this corollary to illustrate the conceptual foundation of our setting. Evolutionary games occur in pre-market settings in which private actors cannot rely on rule of law to enforce contracts. Thus, our Proposition 2 is robust in this more primitive setting.

3 Bargaining through Output Sharing

While evolutionarily stable, our stealing equilibrium generates fights that are socially costly. The costs of fights are pure deadweight losses, which both the producer and

[5]The strategic nature of the players' interactions in our model distinguishes it from evolutionary economic models such as Robson and Samuelson [2009]. A key assumption of any game-theoretic model is that the players are cognitively advanced enough to execute the equilibrium strategies. This assumption in our setting is in accord with modern biology, which argues that complex strategic behavior and the ability to construct mental models of others' motives and behaviors, the two main features of game theory, co-evolved with each other. In fact, Tomasello [2014] argues that the mental modeling capacity in humans has advanced to the point where they can apply the model to themselves to generate "consciousness."

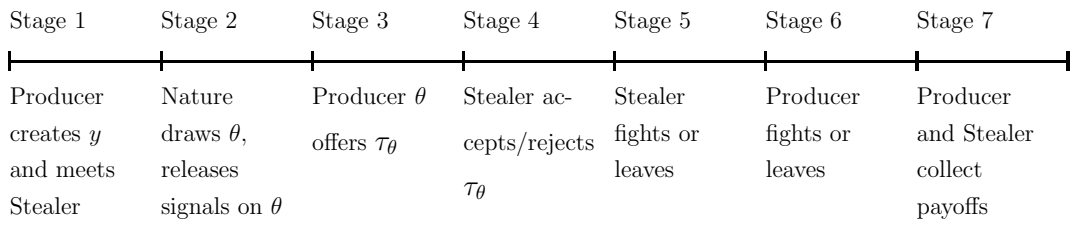

Figure 5: The Sharing Game

stealer would like to avoid. It is, therefore, of interest to see if another equilibrium in a modified game exists that avoids such fights.[6] The Coase Theorem suggests that absent transaction costs, individual parties can bargain towards efficient outcomes. Therefore, it seems plausible that a simple bargaining game would allow the parties to avoid conflict. We propose one simple example of such a bargaining game in this section.

To gain traction on this, we extend the benchmark game to allow the producer to share his output with the stealer after both parties receive their private signals. The producer offers the stealer a deterministic τ_T share of the output when Tough and a deterministic τ_W share of output when Weak, in the hope that the stealer will not engage in a costly fight. Call this extension the "sharing" game, which we illustrate in Figure 5 through a simple timeline. An important feature in this game (and indeed throughout the paper) is that in pre-market settings, no party can commit to any promises. Therefore, even after the producer θ offers τ_θ and the stealer accepts, there is nothing prohibiting the stealer from attacking afterwards. But even in such a setting without commitment, in large measure, there is a simple equilibrium of no sharing:

Proposition 3 *The sharing game has a sequential equilibrium that is unique for every value of y and satisfies the Intuitive Criterion. This equilibrium is $\tau_T = \tau_W = 0$, except*

[6] Our cue for this investigation comes from a similar attempt in the classic resource competition Hawk-Dove game (Smith [1982, Ch.2]). In that game, the Dove strategy is to share with another Dove and retreat in the face of a Hawk strategy. By contrast, the Hawk strategy is to escalate the contest. This game also admits another ESS, namely the "bourgeois" strategy, where the incumbent, or the owner of the resource, always plays aggressively, and the intruder plays passively (Smith [1982, Ch.8]; Gintis [2009, Ch.6]). The bourgeois strategy, in addition to being ESS, has the added advantage that it entails no conflicts.

for small ranges of y that can be made arbitrarily small (by reducing μ_{pT}) or completely eliminated (if q_s is sufficiently high.)

Beliefs are a critical part of the equilibrium because the strategy for the players must be optimal with respect to their beliefs. The proof defines belief restrictions for both pooling and separating equilibria. These are standard restrictions on beliefs from the signaling literature: the belief must be equal to the prior probability if it is a pooling equilibrium and must fully reveal if it is a separating equilibrium.

Proposition 3 emerges from two main properties of the equilibrium. The first property is that there are no separating equilibria: the Weak producer always wants to mimic the Tough producer. The second property is that there are two types of pooling equilibria: the no-sharing equilibrium, which holds for most of the output levels, and positive sharing, that exists for certain output levels near the target thresholds defined in Proposition 3.

The intuition for zero equilibrium transfers is that the Tough producer cannot identify himself as Tough by sharing because the Weak producer mimics the Tough producer's sharing; the Tough producer, therefore, offers the lowest amount, zero. The positive sharing equilibrium over a certain range of y arises not from an informational but from a transactional aspect of the model: the Tough producer's cost of fighting is always μ_{pT} and it may not be worth expending this discrete cost if sharing a small part of the output causes the stealer to discontinuously lower his probability of attack. For example, if y is just above y_W, the Tough producer is better off sharing a small amount and reducing y below the threshold y_W: the stealer never attacks if the output is less than y_W. The proof makes it clear that as μ_{pT} becomes smaller, the range of y worth defending increases all the way to the full range.

Proposition 3 shows the existence of a pooling equilibrium in which both types of the producer will offer the same (zero) share of output in the sharing game. This occurs because the weak producer seeks to distort the statistical inference from the sharing choice. This signal jamming effect forces each party to offer an ever smaller share. To see this, observe that the Tough producer is willing to offer some non-zero share τ_T in order to avoid the costly fight. If the Weak producer offers any share different than this initial proposed share of the strong producer ($\tau_S \neq \tau_T$), then he will reveal his type and invite aggression. As such, the weak producer then matches exactly the initial proposal $\tau_S = \tau_T$. In response to that, the strong producer then reduces his share to $\tau_T' < \tau_T$ in

order to distinguish himself from the weak producer. But if the strong producer offers τ'_T, the weak producer would also seek to offer $\tau'_S = \tau'_T$, again to jam the signal and prevent the stealer from distinguishing types. This process repeats until it arrives at the only remaining equilibrium, the smallest possible share of 0. Thus, every separating equilibrium unravels, leaving only the sole pooling equilibrium that survives.

The ranges $y \leq y_W$ and $y > y_T$ are not interesting because the stealer always retreats or attacks irrespective of his signal. The most important case is when the range of the output $[\underline{y}, \bar{y}]$ is within the range $[y_W, y_T]$. In this case, the stealer attacks if his signal is strong and retreats if his signal is weak. For this important case, the equilibrium is $\tau_T = \tau_W = 0$. The intuition, as stated above, is that the Tough producer cannot signal him type by sharing because the Weak producer mimics the Tough producer. The Tough producer, therefore, offers zero. The proof also makes it clear that a sufficiently precise stealer's signal ensures that $[\underline{y}, \bar{y}]$ is within the range $[y_W, y_T]$, causing this important case to prevail. For the numerical example after Proposition 1, the threshold q_s is $\frac{12}{17}$.

4 The Value of Auditing

The no-sharing result of Proposition 3 implies that no sharing leads to socially wasteful fights. The key economic force behind the robustness of the model's stealing equilibrium is the *information asymmetry* between the producer and the stealer. If this information asymmetry can be reduced, one can generate more cooperative solutions.

This raises the question as to how the complete information situation can be enabled. Consider a solution documented by modern biological research: the use of impartial third parties (von Rohr et al. [2012]). The origins of impartiality presumably have both a biological moral basis and a social basis (i.e., norms, reputations, and other social incentives), reflecting the cognitive skills represented by Kahneman System 2 (also see Trivers [1971]). We illustrate this point by introducing an impartial party who will truthfully reveal the producer's type for an ex ante transfer k from the producer.[7]

To fix ideas, consider an expanded "certification" game based off of the original model in Section 2. Now, after the producer and stealer receive their type, the producer can pay a third party a certification fee k in order to fully reveal him type.

Figure 6 shows the timeline of the certification game, which includes the option

[7]Note that k accrues to the impartial party and is therefore not a social loss like the fight costs.

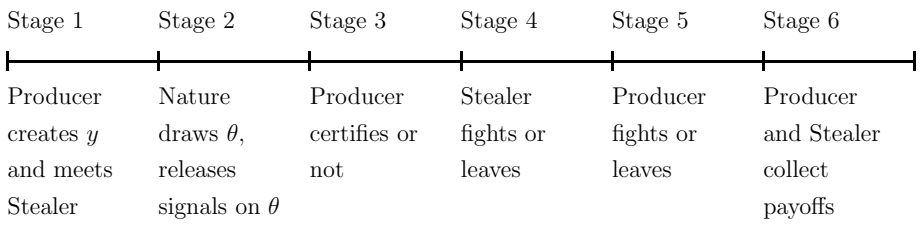

Figure 6: The Certification Game.

for the producer to certify at a fee. There is no moral hazard problem in the model with respect to auditing. Rather, the auditor faces a non-contingent fee and provides flawless certification. In practice, auditors are subject to moral hazard problems just like everyone else. But there is already a large literature on auditor moral hazard, and we seek to focus on the effects of certification on firm behavior. The next proposition characterizes a Tough producer's demand for this third party's services:

Proposition 4 *If the certification fee k is less than him cost of conflict ($k < \mu_{pT}$), the Tough producer certifies, and the no-fight equilibrium ensues.*

Proposition 4 effectively creates a separating equilibrium in which the strong producers certify but the weak do not. Certification for the weak producer would simply reveal his type and, therefore, invite aggression from the stealer. Certification from the strong producer will also reveal his type and forestall aggression for the same reasons. Certification is a means of eliminating the asymmetric information in the game. The certification here is perfect but could easily be extended to imperfect certification. In such a case, the stealer would receive a noisy signal from the auditor, which would have value only if it was more precise than the stealer's own signal. It is a straightforward extension to show that imperfect certification would lead to non-zero fighting in equilibrium but still less fighting than without certification altogether.

The tough producer's willingness to sacrifice a fixed amount ex ante is different from him behavior in Proposition 3. More generally, the producer and the stealer could be part of the same social group that engages repeatedly and whose members care about each other (Wilson [2012]). In such cases, it is likely that the producer and stealer will share the output and not engage in costly infighting. It is in this manner that trust,

information asymmetry, and other group-related factors drive individual behaviors and preferences.

We assume throughout that certification generates hard information that can be verified ex-post, rather than soft information that is subjective and cannot withstand scrutiny in a formal assessment, like a court of law. This assumption is slightly stronger than our original assumption of no commitment but only in that it requires a third party to verify quality. If certification was not possible at all, i.e. if hard information is impossible to establish, then Proposition 4 will not hold, and the costly conflict of Proposition 2 will ensue. Indeed, this is part of the problem in emerging markets where rule of law and property rights are weak. Under such conditions, no one believes a third party's attestation on the value of output, especially if that party is subject to bribes. We effectively assume away that kind of manipulation by proposing that hard information is possible. The combination of Proposition 1 and Proposition 4 suggests that disclosure, when backed by auditing that produces hard information, can eliminate firm conflict. Proposition 4 effectively is a kind of voluntary disclosure in which firms choose whether to disclose their type by certifying with the auditor. The firm does not need to certify (or disclose), and only the strong will elect to do so in order to separate from the weak. Proposition 1 essentially consists of involuntary disclosure since the stealer's signal is accurate and there is no informational difference. Such involuntary disclosure will fully reveal the types in the market and, thereby, forestall conflict. The concern here is not so much of voluntary versus involuntary disclosure but rather on the existence of certification that can be used to break the pooling equilibrium of the incomplete information game.

5 Empirical Implications

Moving forward to modern markets, these results apply to firms who both rely on outside investors for capital as well as who compete with other firms in product markets for customers. Today's firms may have originally provided audited financial statements for the purpose of raising capital, but it is impossible to ignore the effects of that information on their product markets. As information and innovation is central to economic growth and firm profitability, some firms will have a comparative advantage in the production of innovation and others in expropriation. If so, then financial statement information disclosed in the market will not only affect capital investment but also alter the strategic

choices of existing firms. Regulating bodies like the Financial Accounting Standards Board mandate rules for all firms to follow (mandatory disclosure), but firms still have choices on how much to disclose above and beyond those rules (discretionary disclosure). Auditing emerges from the desire to make certain the entire financial statement, which includes both mandatory and voluntary components and, thus, alters the game-theoretic decisions of firms.

Proposition 4 shows the value of certification in mediating conflicts between self-interested firms. Auditing is the chief representation of this certification in modern markets. The implications of a conflict-oriented approach, rather than an investor-oriented approach, are many. First, the natural question is whether certification effects private and public companies differently. The primary manifestation of conflict in market economies is litigation, especially intellectual property litigation. When company A expropriates the ideas or assets of company B, then company A serves as the defendant and company B as the plaintiff in a lawsuit.

Corollary 3 *Controlling for firm size, litigation between private companies exceeds litigation between public companies.*

While data on private company litigation is difficult to obtain, since private companies face no disclosure laws by construction, there is some anecdotal survey evidence that suggests that this corollary may be empirically valid. A more careful empirical test would look at a single corporation before and after its IPO and track whether the amount of litigation it faces markedly decreases after going public.

Of course, there is an important countervailing effect with public companies. They are usually larger, more developed, and have higher value projects and ideas since their need for capital and growth is what drove them to go public in the first place. Thus, public companies may be easier targets for expropriation because of their greater size and quality. Nonetheless, the existing survey evidence suggests that though large company lawsuits are larger in dollar terms than small company lawsuits, the majority of the number of lawsuits from IP litigation worldwide stems from private, not public companies.[8] We now turn to our second implication.

[8] In fact, the industry survey NPE [2015] states that "62% of unique defendants in 2014 had less than $100 million in annual revenue. The only appreciable drop in NPE litigation frequency has occurred for companies with between $10 and $50 billion in annual revenue."

Corollary 4 *Public companies choosing higher levels of disclosure will enjoy less litigation and expropriation.*

Because public companies vary in their level of disclosure, there is cross-sectional variation that can expose the differential effects of auditing. Because most firms comply with mandatory disclosure laws and on top of this select differential levels of voluntary disclosure, our theory predicts that high types will voluntary disclose in order to forestall expropriation and theft. Companies that disclose more provide information to the market that can deter expropriation, whereas opaque companies may invite more expropriation. Thus, our theory on auditing provides implications of financial disclosure on the competitive behaviors of firms against one another.

6 Conclusion

The modern capital market today no doubt includes both investors and competitive firms in the same economy. The prior literature has focused almost exclusively on the problem of information channels to investors and whether shareholders can resolve the auditor's moral hazard problems with an appropriate nexus of threats, penalties, and rewards. In this sense, auditing is cast as a corporate governance problem of protecting investors. Because firms compete with each other in a dynamic and ever changing marketplace, they constantly use financial statement information to make competitive decisions. We have proposed one such angle in which some firms produce and others expropriate. Ours is not the only possible characterization of firm contests but simply one example of how auditing can alter firm dynamics within the same industry.

Our goal is not to downplay the capital market function of auditing but rather to illustrate an alternative explanation. Even though auditing may have emerged primarily out of a capital market need, it functions today in a marketplace where firms routinely compete with one another and, therefore, likely has a role in that competition. Our paper, to our knowledge, is the first to articulate precisely the role of auditing in a competitive market where some firms produce and others expropriate. Regulators, such as the Public Company Accounting Oversight Board, should at least be aware of this game theoretic role of auditing in addition to the more classical attention to the flow of information between auditors and investors.

Future research will provide an even richer set of institutional detail on the relations

between firms, along with more refined choice sets of auditors. For example, what is the role of unaudited disclosures such as quarterly financial statements or press releases? What is the effect of changes in the auditor's legal liability on firm behavior? How does the auditor's internal organization, such as split between audit and non-audit services, affect the firm's productive decisions? How do audit standards, such as the recent push by the PCAOB to disclose audit engagement partner information, affect firm's competitive behavior? This "real effects" research program is an open road.

7 Appendix

Proof of Proposition 1: Suppose the producer is Tough, which occurs with probability ϕ. In this state of the world, work backwards from the last node in the game tree, which is the producer's action. If the stealer fights, the producer earns $y - \mu_{pT} > 0$ by fighting and nothing by leaving, so the producer prefers to fight. The stealer earns $-\mu_{sT}$. The stealer therefore prefers to leave. Thus, the producer earns all the surplus y, and the stealer earns nothing.

Suppose the producer is Weak. If the stealer fights, a Weak producer will lose against the stealer and earn $-\mu_{pW}$ by fighting and 0 by leaving; therefore, the producer will leave. Knowing this, the stealer will fight, earning the full surplus y while the producer gets nothing. This establishes the Nash equilibrium of the extensive form game. The same logic shows it is unique and sub-game perfect.

■

Proof of Proposition 2: Let $a_s(\sigma_s)$ be the stealer's strategy as a function of his signal $\sigma_s \in \{T, W\}$ for T = Tough and W = Weak. Let $a_p(\theta)$ be the producer's strategy as a function of him type $\theta \in \{T, W\}$. Let b be the vector of beliefs at each information set in the extended game. We seek to find the Perfect Bayesian equilibrium $(a_s^*(\sigma_s), a_p^*(\theta))$.

To solve, work backwards. Figure 3 presents the extensive form of the game. Observe that the producer has a nontrivial choice only when the stealer chooses to fight. For a producer of type $\theta = T$, he earns $y - \mu_{pT}$ by fighting and 0 by leaving, regardless of the stealer's signal. Since $y > \mu_{pT}$ by assumption, $a_p^*(T) = F$. Similarly, a Weak producer makes a choice only when the stealer chooses to fight. This producer earns 0 by leaving and $-\mu_{pW}$ by fighting, regardless of the stealer's signal. Thus $a_p^*(W) = L$. This proves parts 3 and 4.

To solve for the stealer's strategy, it is necessary to compute the stealer's beliefs according to Bayes' rule. Let b_{ij} denote the stealer's belief that $\theta = i$ when the stealer's signal $\sigma_s = j$. Calculated by Bayes' rule, this yields:

$$b_{TT} = \frac{\phi q_s}{\phi q_s + (1-\phi)(1-q_s)}, \tag{8}$$

$$b_{TW} = \frac{\phi(1-q_s)}{\phi(1-q_s) + (1-\phi)q_s}, \tag{9}$$

$$b_{WT} = \frac{(1-\phi)(1-q_s)}{\phi q_s + (1-\phi)(1-q_s)}, \tag{10}$$

$$b_{WW} = \frac{(1-\phi)q_s}{\phi(1-q_s) + (1-\phi)q_s}. \tag{11}$$

Suppose $\sigma_s = W$. Given $a_p^*(\theta)$, if the stealer fights, he earns: $\Delta_W = b_{WW}y - b_{TW}\mu_{sT}$. He earns nothing if he leaves. Thus, $a_s^*(W) = F$ if and only if $\Delta_W > 0$, and $a_s^*(W) = L$ otherwise.

Now suppose $\sigma_s = T$. Given $a_p^*(\theta)$, the stealer earns 0 if he leaves and if he fights, he earns, in expectation, $\Delta_T = b_{WT}y - b_{TT}\mu_{s_T}$. Thus, $a_s^*(T) = F$ if and only if $\Delta_T > 0$, and $a_s^*(T) = L$ otherwise. This proves parts 1 and 2.

To see uniqueness, first note that the producer's equilibrium strategy in the action space of Figure 3 is strictly dominant. Given the producer's strategy, the conditions for $\Delta_W > 0$ and $\Delta_T > 0$ are necessary and sufficient to determine the stealer's best response. The equilibrium is thus unique.

■

Proof of Corollary 1: Note from (1) that $f(y;e)$ is linear in e; its second derivative with respect to e is, therefore, zero. Because the integrand and its e derivatives are piecewise integrably bounded, Leibniz's rule applies to this integral, and shows that:

$$\frac{\partial^2}{\partial e^2}\left[\int_{\underline{y}}^{\bar{y}} B(y)f(y;e)\,dy - C(e)\right] = \int_{\underline{y}}^{\bar{y}} B(y)\frac{\partial^2}{\partial e^2}f(y;e)\,dy - C''(e) = -C''(e) < 0. \tag{12}$$

The optimization problem of the Proposition with respect to e is therefore strictly concave and yields a unique solution for the optimal e. This unique solution is less than $\bar{e}$ because $C'(\bar{e}) = \infty$. The presence of discontinuous drops in $B(y)$ implies that we cannot assert that the optimal e is always greater than $\underline{e}$. However, the possibility that the optimal e can equal $\underline{e}$ has no impact on our analysis. Moreover, there are a wide variety of settings in which the optimal $e > \underline{e}$. To see this, this first note that $\underline{y}, \bar{y}$ are assumed to be greater than μ_{pT}, whereas y_W, y_T do not depend on μ_{pT} but on the other free exogenous parameter μ_{sT}. As a result, any of the following three possibilities, among others, are settings where we can show that the optimal $e > \underline{e}$: 1) $\underline{y} < \bar{y} < y_W$

(the stealer never attacks), 2) $y_W < \underline{y} < \bar{y} < y_T$ (the stealer attacks only if he receives the Weak signal), and 3) $\underline{y} > y_T$ (the stealer always attacks). In each case, $B(y)$ is a linear function with positive slope. Calculation shows that $\frac{\partial}{\partial e}\int_{\underline{y}}^{\bar{y}} B(y)f(y;e)\,dy\Big|_{e=\underline{e}} > 0$, and $C'(\underline{e}) = 0$. So, $e = \underline{e}$ cannot be the solution in any of these cases.

■

Proof of Corollary 2: We follow the definition of ESS from Selten [1980]. Every strict equilibrium is ESS. As a refinement, the ESS criterion is useful for selecting among multiple strategies that are possible best responses to the equilibrium strategy. When the pure equilibrium is strict, as is the case here, there is only a single best response to the equilibrium strategy, namely, the equilibrium strategy itself. The action space is given in Figure 3.

It remains to show that the equilibrium is strict. Recall that a strategy is a mapping from each agent's signal to an action F or L. Let $a_p^*(\theta)$ and $a_s^*(\sigma_s)$ denote the equilibrium strategy for the producer and stealer, respectively, based on their signals, θ and σ_s. It is necessary to show that each agent's equilibrium strategy is a strict best response to the other agent's equilibrium strategy. For the producer, this means that each for θ,

$$EU_p(a_p^*(\theta), a_s^*(\sigma_s)) > EU_p(a_p, a_s^*(\sigma_s)), \tag{13}$$

$\forall a_p \neq a_p^*(\theta)$, where the inequality is strict. The expectation is taken with respect to posterior beliefs computed by Bayes' rule. For the stealer, a strict best response must satisfy for each σ_s,

$$EU_s(a_p^*(\theta), a_s^*(\sigma_s)) > EU_s(a_p^*(\theta), a_s), \tag{14}$$

$\forall a_s \neq a_s^*(\theta)$. Consider the equilibrium strategy from Proposition 2. A Tough producer fights and a Weak producer leaves, so $a_p^*(T) = F$ and $a_p^*(W) = L$. For the producer, this strategy is in fact a dominant strategy. To show that it is strict, observe that the Tough producer fights if and only if $y > \mu_{pT}$. Similarly, a Weak producer leaves if and only if $0 < \mu_{pW}$. Since these inequalities are strict by assumption, him equilibrium strategy is therefore a strict best response.

Now consider the stealer's strategy. There are three possible cases. In each case, the producer's strategy is the same at $a_p^*(T) = F$ and $a_p^*(W) = L$.

First, consider the case $a_s^*(T) = L$ and $a_s^*(W) = F$. This occurs if and only if $\Delta_T < 0$ and $\Delta_W > 0$ (we ignore the zero measure realization of y where the second inequality becomes an equality). Since these inequalities are strict, the stealer's equilibrium strategy is a strict best response.

Consider next the case $a_s^*(T) = F$ and $a_s^*(W) = F$. This occurs if and only if $\Delta_T > 0$ (we ignore the zero measure realization of y which makes this inequality an equality). Since this inequality is strict, the stealer's best response is also strict. Finally, consider the third case where the stealer always leaves: $a_s^*(T) = L$ and $a_s^*(W) = L$. From Proposition 2 this occurs if and only if $\Delta_W < 0$ and $\Delta_T < 0$. Since these inequalities are strict, the stealer's best response is also strict. Thus, the Perfect Bayesian equilibrium is strict and, therefore, ESS. ■

Proof of Proposition 3: Suppose the producer sees the stealer and learns him own type (Tough or Weak). The expected payoff to the Tough producer after meeting the stealer, but before the fight, is:

$$B_T(y) = \begin{cases} y & : y \leq y_W \\ y - (1 - q_s)\mu_{pT} & : y_W < y \leq y_T \\ y - \mu_{pT} & : y > y_T. \end{cases} \tag{15}$$

The expected payoff to the Weak producer after meeting the stealer, but before the fight, is:

$$B_W(y) = \begin{cases} y & : y \leq y_W \\ (1 - q_s)y & : y_W < y \leq y_T \\ 0 & : y > y_T. \end{cases} \tag{16}$$

Note that because $q_s > \frac{1}{2}$ and $y > \mu_{pT}$, $\forall y : B_T(y) \geq B_W(y)$.

Suppose the producer can offer to share a part of the output in the hope that the stealer will leave. The stealer receives the transfer, observes the size of the rest of the output, and then gets his signal about the producer's type. He then decides whether to continue the stealing game for the rest of the output. We solve for an equilibrium in this game.

Separating Equilibrium: First, consider separating equilibria. Let (t_W, t_T) be the equilibrium transfers offered in a separating equilibrium by the Weak and the Tough

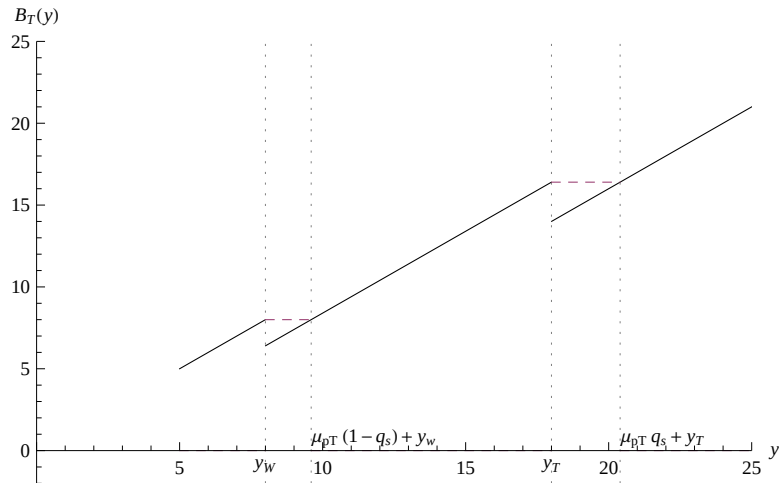

Figure 7: The benefit function $B_T(y)$ for $m = 0.75$, $\phi = 0.5$, $q_s = 0.6$, $\mu_{pT} = 4$, and $\mu_{sT} = 12$, $y \in [5, 25]$. The horizontal lines are the regions where the Tough producer shares to increase him payoff.

producer respectively. Let $P(t)$ be the stealer's probability assessment that the producer is the Tough type. By consistency of beliefs, $P(t_W) = 0$ and $P(t_T) = 1$. Thus, by submitting t_W, the Weak producer gets 0 (the stealer will attack for the rest of the output for sure), whereas by deviating to t_T and pooling with the Tough producer, he will be weakly better off than zero (we will show that he earns $B_W(\cdot)$, which from equation (16) is weakly greater than zero). This deviation is profitable and, therefore, no separating equilibria exist, i.e., $t_W = t_T$.

We next determine the optimal $t_W = t_T$ and show that it is unique in y. We first specify the off-equilibrium beliefs of the stealer:

Pooling Equilibrium's Off-Equilibrium Beliefs: Given that the equilibrium $t_W = t_T$ is pooling, we assume that $P(t) = \phi$ for any feasible t. That is, the transfer itself does not change the priors of the stealer. We will prove later that our equilibrium based on this belief structure satisfies the intuitive criterion.

We consider the Tough producer's case first. This producer offers t_T such that:

$$t_T = \arg\max_{0 \le t \le y} B_T(y - t). \tag{17}$$

This recursive formulation occurs because, by assumption, an offering of t does not change the stealer's priors from ϕ. Therefore, for the remaining output $y - t$, for all

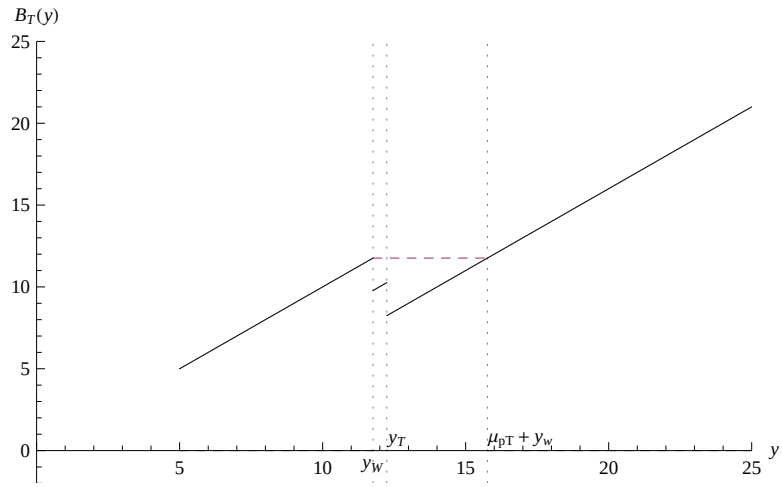

Figure 8: The benefit function $B_T(y)$ for $m = 0.75$, $\phi = 0.5$, $q_s = 0.505$, $\mu_{pT} = 4$, and $\mu_{sT} = 12$, $y \in [5, 25]$. The horizontal line is the region where the Tough producer shares to increase him payoff.

feasible t, the stealer acts sequentially rationally and the Tough producer expects to get, by definition of $B_T(.)$ above, $B_T(y - t)$.

If $B_T(y)$ were monotonically increasing, $t_T = 0$. But there are regions where it is not. Given the structure of the exogenous variables, there are two cases to consider: $B_T(y_W) \leq B_T(y_T)$ and $B_T(y_W) > B_T(y_T)$. We solve for each case separately.

Case I: $B_T(y_W) \leq B_T(y_T)$

This condition implies that:

$$\begin{aligned} & y_W \leq y_T - (1 - q_s)\mu_{pT} \\ \text{or } & \mu_{pT} \leq \frac{y_T - y_W}{1 - q_s} = \frac{\phi(2q_s - 1)}{(1 - \phi)(1 - q_s)^2 q_s}. \end{aligned} \tag{18}$$

Figure 7 plots an example $B_T(y)$. Note in this figure that $\mu_{pT} = 4 < \frac{18-8}{1-0.6} = 25$.

The smaller local maximum of $B_T(y)$ is attained at $B_T(y_W)$, and the larger local maximum is attained at $B_T(y_T)$. We also note that:

$$B_T(y_W) = y_W > B_T(y), y \in (y_W, y_W + (1 - q_s)\mu_{pT}). \tag{19}$$

So, for $y_W < y \leq y_W + (1 - q_s)\mu_{pT}$, the Tough producer will make a transfer $t_T = y - y_W$ and get the payoff y_W. The Weak producer will mimic that transfer and

also earn y_W because the stealer will not attack when remaining output is less than or equal to y_W. Further note that in this case, $y_W + (1 - q_s)\mu_{pT} \in (y_W, y_T]$, the middle continuous range of $B_T(y)$.

We also see from equation (15) that:

$$B_T(y_T) = y_T - (1 - q_s)\mu_{pT} > B_T(y), y \in (y_T, y_T + q_s\mu_{pT}). \tag{20}$$

So, for $y_T < y \leq y_T + q_s\mu_{pT}$, the Tough producer will transfer $t_T = y - y_T$ and get the payoff $B_T(y_T)$. The Weak producer will mimic that transfer and earn $(1 - q_s)y_T$. Note that because $B_T(y_T) \geq B_T(y_W)$, by assumption, the Tough producer will not transfer the larger amount $y - y_W$.

The complete specification of the equilibrium for Case I is:

1. Set the default $t_T = t_W = 0$. Then apply the following changes in order.
2. If $y \leq y_W$, $t_T = t_W = 0$.
3. If $y_W < y \leq y_W + (1 - q_s)\mu_{pT}$ and y_W is feasible (i.e., $y_W > \underline{y}$), then $t_T = t_W = y - y_W$. If y_W is not feasible, then $t_T = t_W = 0$.
4. If $y_T < y \leq y_T + q_s\mu_{pT}$ and y_T is feasible (i.e., $y_T > \underline{y}$), then $t_T = t_W = y - y_T$. If y_T is not feasible, then $t_T = t_W = 0$.
5. The Tough producer earns $B_T(y - t_T)$, and the Weak producer earns $B_W(y - t_W)$.

Case II: $B_T(y_W) > B_T(y_T)$

This condition implies that:

$$\begin{aligned} & y_W > y_T - (1 - q_s)\mu_{pT} \\ \text{or } & \mu_{pT} > \frac{y_T - y_W}{1 - q_s} = \frac{\phi(2q_s - 1)}{(1 - \phi)(1 - q_s)^2 q_s} > 0. \end{aligned} \tag{21}$$

Figure 8 plots an example $B_T(y)$ for the Case II scenario. The only change in the parameters from Figure 7 is the reduction in the accuracy of the stealer's signal q_s from 0.6 to 0.505. The smaller local maximum of $B_T(y)$ is attained at $B_T(y_T)$, and the larger local maximum is attained at $B_T(y_W)$. If the Tough producer's current payoff is less than these maxima, the Tough producer will move to the relevant maximum.

The complete specification of the equilibrium for Case II is:

1. Set the default $t_T = t_W = 0$. Then apply the following changes in order.

2. If $y \leq y_W$, $t_T = t_W = 0$.

3. If $y_W < y \leq y_T$ and y_W is feasible (i.e., $y_W > \underline{y}$), then $t_T = t_W = y - y_W$. If y_W is not feasible, then $t_T = t_W = 0$.

4. If $y_T < y \leq y_W + \mu_{pT}$ and y_W is feasible (i.e., $y_W > \underline{y}$), then $t_T = t_W = y - y_W$. (Note that $y_W + \mu_{pT} > y_T - (1 - q_s)\mu_{pT} + \mu_{pT} > y_T$.)

5. If y_W is not feasible but y_T is, then, as in Case I, if $y_T < y \leq y_T + q_s\mu_{pT}$ then $t_T = t_W = y - y_T$, and if $y > y_T + q_s\mu_{pT}$ then if $t_T = t_W = 0$.

6. If y_T is not feasible, then $t_T = t_W = 0$.

7. The Tough producer earns $B_T(y - t_T)$, and the Weak producer earns $B_W(y - t_W)$.

Note that in Case II, the equilibrium $t_T = t_W = 0$ when $y > y_W + \mu_{pT}$: the Tough producer gains more at y than at y_W.

Finally, we observe several aspects of the equilibrium common to both Cases:

1. The equilibrium $t_T = t_W$ is unique in y in both Case I and Case II.

2. The equilibrium $t_T = t_W$ is 0 if the range $[\underline{y}, \bar{y}]$ is such that $B_T(y)$ is continuous in the entire range.

3. As $q_s \to 1$, $y_W \equiv \frac{(1-q_s)\phi}{q_s(1-\phi)}\mu_{sT} \to 0$ monotonically, and $y_T \equiv \frac{q_s\phi}{(1-q_s)(1-\phi)}\mu_{sT} \to +\infty$ monotonically. The limiting values of y_W, y_T also imply that for any given admissible values of $\underline{y}, \bar{y}$, there exists a threshold $\bar{q_s}$, $\frac{1}{2} < \bar{q_s} < 1$, above which $y_W < \underline{y} < \bar{y} < y_T$, thus resulting in the equilibrium $t_T = t_W = 0$.

4. In both Case I and Case II, the regions where $t_T = t_W > 0$ are controlled by the size of μ_{pT}. As μ_{pT} shrinks, the chances of non-zero $t_T = t_W$ shrinks. (See the horizontal regions and their boundaries in Figures 7 and 8.)

5. With probability 1, a random choice of exogenous parameters will admit a range of y where $t_T = t_W = 0$. To see this, the probability that $\underline{y} = y_W$ is 0. If $\underline{y} < y_W$, then there exists an $\epsilon > 0$ such that the optimal t_T for $y \in (y_W - \epsilon, y_W)$ is 0. A similar $(\underline{y}, \underline{y} + \epsilon)$ range can be found if $\underline{y} > y_W$.

Intuitive Criterion: We argue that our equilibrium satisfies the intuitive criterion. Recall that a (sequential) equilibrium fails to satisfy the intuitive criterion if, for some non-equilibrium transfer t, the following condition is met: for all possible beliefs of the stealer upon receiving the non-equilibrium t, the Weak (Tough) producer is worse off than in equilibrium, and therefore, the stealer infers upon receiving t that the producer is Tough (Weak), and this inference is reinforcing in that, because of this inference, the Tough (Weak) producer is better off than in equilibrium, and so the Tough (Weak) producer will deviate to the non-equilibrium t (Kreps [1990, p.436, p.818]).

In our model, consider a t that makes the Tough producer worse off (compared to the equilibrium) even when the stealer does not attack. The Tough producer will not issue this t because it is strictly equilibrium dominated. A Weak producer will not issue this t either, for the stealer will put probability one that he is Weak and attack for sure. This gives him 0, and we have shown that the he is weakly better off than 0 in equilibrium.

On the other hand, suppose there is an off-equilibrium t that makes the Weak producer worse off (compared to equilibrium) even in the best-case possibility that the stealer receiving this t does not attack. That is, $y - t$ is less than what he earns earns in equilibrium. This implies that the stealer, upon receiving t, infers that the producer is definitely Tough. We have shown, using equations (15) and (16), that $\forall y : B_T(y) \geq B_W(y)$, so the Tough producer earns weakly more than the Weak producer in the pooling equilibrium. Therefore, the best the Tough producer can do by deviating to t is $y - t$ (i.e., when the stealer does not attack), which is less than what the Weak producer earns in equilibrium, which is less than what the Tough producer earns in equilibrium. So the Tough producer will not deviate to the off-equilibrium t.

A final remaining point is to show that our equilibrium is sequential. Consider the sequence $0 < \{\epsilon_n\} < 1$ where both the Tough and Weak producers issue the equilibrium t^* with probability $1 - \epsilon_n$, and issue all other $t \in [0, y] - \{t^*\}$ using a uniform distribution of total probability measure ϵ_n. This is a strictly mixed sequence of actions with strictly positive Bayes-consistent beliefs, i.e., the stealer's posterior about the Tough type after receiving any t is still ϕ, which is strictly positive because $0 < \phi < 1$. This sequence's limit $\epsilon_n \to 0$ yields our equilibrium actions and beliefs, and this equilibrium, by construction, is sequentially rational. Our equilibrium satisfies the intuitive criterion.

■

Proof of Proposition 4: We seek to establish the certification threshold k^* such that the producer will certify rather than play the game. Consider the benchmark model illustrated in Figure 2. Call the equilibrium in the benchmark model in Proposition 2 the "benchmark equilibrium."

Consider the "certification game." In this game, after Nature reveals θ to the producer, a third party can reveal θ to the stealer for a certification fee of k, which the producer pays. If the producer agrees to certify, then the stealer's revised signal is noiseless and $\hat{q}_s = 1$. If the producer does not certify, then the game proceeds as in the benchmark model.

To solve this, work backwards. Recall that in each of the benchmark equilibria, $a_p^*(T) = F$ and $a_p^*(W) = L$. In every equilibrium, a Weak producer will never choose to certify since the stealer will always fight if he knows the producer is Weak; the Weak producer would earn zero by certifying and earns ≥ 0 by not certifying since not certifying may lead the stealer to leave in some equilibria. The Weak producer has no incentive to certify since there is no benefit to revealing his type.

Consider the Tough producer. Recall there are three equilibria in which the stealer's strategy varies with Δ_T and Δ_W. We will consider these equilibria case by case.

First suppose $\Delta_T < 0$ and $\Delta_W < 0$. In the benchmark equilibrium, $a_s^*(T) = a_s^*(W) = L$. The Tough producer earns $B(y)$, him maximum possible payment, so he will never pay any certification fee.

Suppose $\Delta_T > 0$ and $\Delta_W > 0$. In the benchmark equilibrium, $a_s^*(T) = a_s^*(W) = F$. The Tough producer's equilibrium payoffs are $B(y) - \mu_{pT}$. Rolling the game backwards, if the producer chooses to certify, he pays a certification fee k and $\hat{q}_s = 1$, so the stealer then leaves and the producer collects the full surplus $B(y)$. A Tough producer will do this if he is better off than him payoff in the benchmark equilibrium, or

$$B(y) - \mu_{pT} < B(y) - k. \tag{22}$$

This holds for all $k < k^* = \mu_{pT}$. The certification threshold k^* is the most the Tough producer will pay to certify, in this case equal to him deadweight loss of conflict.

Suppose $\Delta_T < 0$ and $\Delta_W > 0$. In the benchmark equilibrium, $a_p^*(T) = L$ and $a_p^*(W) = F$. In this equilibrium, the Tough producer earns $B(y)$ with probability q_s and $B(y) - \mu_{pT}$ with probability $1 - q_s$. If he certifies, then $\hat{q}_s = 1$ and the stealer will leave, passing the full surplus to the producer minus his certification fee. Thus, the Tough producer will certify if:

$$q_s B(y) + (1 - q_s)(B(y) - \mu_{pT}) < B(y) - k. \tag{23}$$

Rearranging this yields the certification threshold

$$k^{**} = (1 - q_s)\mu_{pT}. \tag{24}$$

Observe that $k^{**} < k^{*}$ since the expected cost of conflict is lower in this equilibrium as the stealer only fights if he gets a Weak signal.

■

References

"NPE Litigation Report." Rational patent, RPX Corporation, 2015.

Apple. "Apple Incorporated 2014 Annual Report." Annual report, Apple Incorporated, 2014.

Chan, D.; and S. Pae. "An analysis of the economic consequences of the proportionate liability rule." *Contemporary Accounting Research* 15 (1998): 457480.

Cosmides, L.; and J. Tooby. "Evolutionary Psychology: A Primer.", 1997.

Deng, M.; N. Melumad; and T. Shibano. "Auditors' liability, investments, and capital markets: A potential unintended consequence of the Sarbanes-Oxley Act." *Journal of Accounting Research* 50 (2012): 11791215.

Dye, R. "Auditing standards, legal liability, and auditor wealth." *Journal of Political Economy* 101 (1993): 887914.

Gerakos, J.; and C. Syverson. "Competition in the audit market: Policy implications." *Journal of Accounting Research* .

Gintis, H. *Game Theory Evolving.* Princeton University Press, 2009.

Hillegeist, S. "Financial reporting and auditing under alternative damage apportionment rules." *The Accounting Review* 74 (1999): 347369.

Kahneman, D. *Thinking, Fast and Slow.* Farrar, Straus and Giroux, 2011.

Kanodia, C.; and A. Mukherji. "Audit pricing, lowballing and audit turnovers: a dynamic analysis." *The Accounting Review* 69 (1994): 593615.

Kreps, D. *A Course in Microeconomic Theory.* Princeton University Press, 1990.

La Porta, R.; F. L. de Silanes; A. Shleifer; and R. W. Vishny. "Legal Determinants of External Finance." *The Journal of Finance* 52 (1997): 49–100.

La Porta, R.; F. L. de Silanes; A. Shleifer; and R. W. Vishny. "Law and Finance." *Journal of Political Economy* 106 (1998): 49–100.

Laux, V.; and P. Newman. "Auditor liability and client acceptance decisions." *The Accounting Review* 85 (2010): 261285.

Liu, C.; and T. Wang. "Auditor liability and business investment." *Contemporary Accounting Research* 23 (2006): 10511071.

Magee, R.; and M. Tseng. "Audit pricing and independence." *The Accounting Review* 65 (1990): 315336.

Melumad, N.; and L. Thoman. "On auditors and the courts in an adverse selection setting." *Journal of Accounting Research* 28 (1990): 77120.

Morgan, J.; and P. Stocken. "The effects of business risk on audit pricing." *Review of Accounting Studies* 3 (1998): 365385.

Narayanan, V. G. "An analysis of auditor liability rules." *Journal of Accounting Research* 32 (1994): 3959.

Newman, D.; E. Patterson; and R. Smith. "The influence of potentially fraudulent reports on audit risk assessment and planning." *The Accounting Review* 76 (2001): 5980.

Newman, D.; E. Patterson; and R. Smith. "The Role of Auditing in Investor Protection." *The Accounting Review* 80 (2005): 289313.

Nowak, M.; and K. Sigmund. "Evolution of Indirect Reciprocity." *Nature* 437 (2005): 1291–1298.

Pae, S.; and S. Yoo. "Strategic interaction in auditing: An analysis of auditors' legal liability, internal control system quality, and audit effort." *The Accounting Review* 76 (2001): 333356.

Parker, G. "Assessment Strategy and the Evolution of Fighting Behavior." *Journal of Theoretical Biology* 47 (1974): 223–243.

Patterson, E.; and D. Wright. "Evidence of fraud, audit risk and audit liability regimes." *Review of Accounting Studies* 8 (2003): 105131.

Porta, R. L.; F. L. de Silanes; A. Shleifer; and R. W. Vishny. "Investor protection and corporate governance." *Journal of Financial Economics* 58 (2000): 3–27.

Radhakrishnan, S. "Investors' recovery friction and auditor liability rules." *The Accounting Review* 74 (1999): 225240.

Robson, A.; and L. Samuelson. "The Evolution of Time Preference with Aggregate Uncertainty." *American Economic Review* 99 (2009): 1925–1953.

Sapolsky, R. *Why Zebras Don't Get Ulcers*. Henry Holt & Co., 1998.

Schwartz, R. "Legal Regimes, Audit Quality, and Investment." *The Accounting Review* 72 (1997): 385–406.

Selten, R. "A Note on Evolutionarily Stable Strategies in Asymmetric Animal Conflicts." *Journal of Theoretical Biology* 84 (1980): 93–101.

Shleifer, A.; R. LaPorta; and F. L. de Silanes. "Law and Finance." *Journal of Political Economy* 106 (1998): 1113–1155.

Smith, J. M. *Evolution and the Theory of Games*. Cambridge University Press, 1982.

Smith, J. M.; and G. Parker. "The Logic of Asymmetric Contests." *Animal Behavior* 24 (1976): 159–175.

Smith, M. J.; and G. R. Price. "The Logic of Animal Conflict." *Nature* 246 (1973): 15–18.

Thoman, L. "Legal damages and auditor efforts." *Contemporary Accounting Research* 13 (1996): 275306.

Tomasello, M. *A Natural History of Human Thinking*. Harvard University Press, 2014.

Trivers, R. "The Evolution of Reciprocal Altruism." *The Quarterly Review of Biology* 46 (1971): 35–57.

von Rohr, C.; et al. "Impartial Third-Party Interventions in Captive Chimpanzees: A Reflection of Community Concern." *PLoS ONE* 7 (2012): e32,494.

Wilson, E. *The Social Conquest of Earth*. Liverlight Press, 2012.