Optimal Performance Targets*

KOROK RAY
Texas A&M University
korok@tamu.edu

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Abstract

I study a class of contracts that is becoming ever more common among executives, in which the manager earns a discrete bonus if his performance clears an explicit threshold. These performance targets provide the firm with an additional instrument to resolve its moral hazard and adverse selection problems with its manager. The performance target can achieve first best under risk neutrality, with a target precisely equal to the desired effort that the firm seeks to induce. The optimal bonus increases in risk. If the manager is risk averse, the firm will shade the optimal target below equilibrium effort to provide a form of insurance to the manager, outside of the standard reduction in the bonus. When the manager has private information on his ability, the optimal bonus and target both increase in ability, to discourage misreporting of his private information.

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1 Introduction

Ever since the Securities and Exchange Commission required companies to disclose details of their executive pay plans in 2006, we now know that many, if not most, companies use some kind of a performance target, in which the manager earns a bonus if his performance clears an explicit threshold. However, the theoretical literature has either examined the class of linear contracts under normally distributed errors and exponential utility (the so-called LEN model), or has articulated optimal nonlinear contracts in full generality that bear little resemblance to contracts used in practice. What is missing is a theoretical exploration of performance targets, motivated by their actual use, to provide both positive prediction and normative guidance.

Until now. I examine the class of performance target contracts under a variety of settings. To fix ideas, I begin with a risk-neutral agent, and show that performance targets achieve first-best, with the target optimally set to efficient effort in an optimal contract. This result bears similarity to the efficiency of rank-order tournaments. And for good reason, since a performance target is like a tournament, except that the target is not a strategic choice by a separate agent, but rather an optimal choice of the firm. The target provides an extra contract parameter, so that the firm can keep one of the other compensation parameters (the bonus) fixed. Thus, the target offers the firm an additional instrument to resolve the manager’s effort problem, a theme that will permeate the analysis. Contrast this to this linear contract, in which salary and bonus both change when the environment changes.

I then examine risk. The canonical model predicts that increases in risk will result in smaller bonuses, as the bonus loads risk onto the manager. However, the empirical evidence on the risk-incentives trade-off has been mixed (Prendergast 2002). Under a performance target, the optimal bonus increases in variation in the manager’s performance measure. When output variance increases, this dampens incentives to work, as it is less likely that output from a given unit of effort will clear the target (because of the increased noise in the system). To compensate for this, the firm increases the bonus in order to extract effort out of the manager. That the bonus increases in noise may help explain the mixed empirical tests of the risk incentives trade-off. These tests largely regress pay-performance sensitivity (PPS) on stock return volatility, and measure PPS through changes in total direct compensation. Here, an increase in risk directly increases the bonus, which will increase direct compensation and therefore PPS, providing a par-
tial explanation for why the empirical tests of the risk incentives trade-off have been mixed.

Next, I solve the model under general risk aversion. While it would be efficient to pay the manager a flat wage in order to provide full insurance, this would ruin incentives to work. I show that the firm will optimally select a target below the second-best equilibrium effort level. Just as the second-best program involves a smaller bonus to reduce the manager’s exposure to risk, so does the lower target provide this insurance effect to the manager. Once again, the target serves as a substitute instrument for the bonus, as they alternatively resolve the manager’s moral hazard problem with the manager.

There will always be two solutions that induce the same effort, given by a low target and a high target. However, even though both targets implement the same effort, the firm is not indifferent. The low target is easier to achieve, and therefore, the manager is more likely to receive his bonus, so his expected bonus compensation is higher. Because of this, he will accept a smaller salary to participate. Because of risk aversion, the firm can lower the bonus also to match incentives at the high target. As such, the firm prefers the low-target contract because it can induce identical effort at lower cost.

I focus attention on finding the optimal contract within the class of performance target contracts; I do not solve for the optimal contract under all possible contracts to show that performance targets are globally optimal. My paper is in the spirit of the LEN literature, which seeks to discover optimal linear contracts within the smaller class of LEN contracts. This focus on a restricted subset of the full contract space has provided much of our core base of knowledge on various incentive schemes and their optimal attributes, such as the risk-incentives trade-off. I depart from the LEN model in my focus on targets (not linear), general forms of risk aversion (not only exponential) and general distributions that are symmetric and single-peaked (not only normal). I write in the spirit of Ross (2004), who urges research to consider properties of contracts that are used in practice, rather than focusing attention exclusively on fully general contracts that are mathematically complex but lack realism.

There’s a small empirical literature on performance targets and an even smaller theoretical one. Murphy (2001) is an early empirical analysis of performance targets that finds that internally-determined performance standards are more likely to have discontinuous features that lead to income smoothing. Murphy (2001) considers compensation in the form of $s + b(X - \bar{X})$, where $X$ is the manager’s performance measure, and $\bar{X}$
is the standard that the manager faces. While this does capture the flavor of a performance that must exceed a standard, it is nonetheless a linear contract in $X - \bar{X}$. Indeed, much of the prior literature assumes targets of this form, and does not model the discontinuous nature of the target explicitly. This paper aims to use the description of performance targets and standards from Murphy (2001), but to model the manager’s optimization problem more explicitly.

Murphy (2001) further documents the presence of an “incentive zone” in which the manager’s pay is linear within the incentive zone, and flat outside of it. I do not consider the optimal incentive zone, as the model itself has enough complexity as is, without examining a linear region in between two different targets. Matejka and Ray (2014) examine the incentive zone in a model of multiple performance targets and differential incentive weights. Gutiérrez Arnaiz and Salas-Fumás (2008) show that the incentive zone collapses to a “dichotomous bonus” (the kind I consider here) when the performance horizon collapses, say from an annual basis to a quarterly basis.

Gutiérrez Arnaiz and Salas-Fumás (2008) solve for the optimal contract in a specific setting. They do not answer the more general question of optimal targets under any symmetric distribution. Their contract is curve-linear in the incentive zone, as it is a function of the likelihood ratio, a standard feature of optimal contracts under risk aversion. This function is convex and then becomes concave after it hits an inflection point, which the authors argue is effectively the performance target. However, because they solve their model in a general continuous framework, they do not have a precise characterization of the optimal bonus and target.

Other theoretical work on targets examines stage financing in venture capital (Dahiya and Ray 2012) and performance evaluation over multiple periods (Ray 2007). Indjejikian et al. (2014) and Gerakos and Kovrijnykh (2013) both consider earnings targets, and the latter paper indeed contains a formal model. However, none of these papers solve for the optimal target. There is of course a large literature on the ratchet effect (Weitzman (1980), Indjejikian et al. (2014), Aranda et al. (2014), Arnold and Artz (2015), and Bouwens and Kroos (2011)), which primarily concerns dynamic changes in targets over time. These papers often ask whether the ratchet effect exists at all, and generally takes

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1 They assume a Symmetric Variance Gamma (SVG) distribution, and the agent makes a one shot change to the mean of a stochastic process. Madan and Seneta (1987) and Carr, Geman, Madan, and Vor (2002) show that the SVG process fits data from share prices, but as of yet, there is no evidence that SVG fits data from accounting numbers, on which most performance targets are based.
the first period target as given, rather than solving for it optimally.

The paper proceeds as follows. Section 2 considers the base model under risk neutrality and discusses the risk-incentives trade-off. Section 3 introduces managerial risk aversion under general utility functions. Section 4 considers adverse selection when the manager has private information, and section 5 concludes.

## 2 The Model

To motivate the model, consider some sample executive pay contracts curated from proxy statements of corporate filings. In 2010, McDonald’s set a target for operating income at $7.24 billion. If the CEO hit this target, his payout was $2,160,000. This target was discrete in that it offered a fixed cash payment if the performance cleared the target and nothing otherwise. Other companies have imposed similar discrete targets. For example, Bank of America in 2011 set a 3-year average ROA target, awarding 33% of the total bonus if the executive’s actual ROA exceeded 50 bps, and nothing otherwise. Barnes and Noble enforced an adjusted EBIT target of -$178.27 million, a low bar given the digital business was expected to have significant cash flow requirements in Fiscal 2014. Chevron set a target based on invested capital with no performance shares awarded if ROIC fell below 18%, 8% awarded if it exceeded 18%, 40% awarded if it exceeded 20%, and 80% awarded if it exceeded 22% or higher. Roughly 38% of the CEO’s compensation was paid in performance shares, delivered in cash.

These are all examples of sample executive contracts that contain some kind of discrete performance target, in which performance must clear an explicit threshold. Often, such targets operate at the low end of performance, in which the executive must obtain a minimal level of performance in order to even receive any kind of payout at all. Sometimes the payout rises linearly with performance, in which case the board interpolates a bonus number for performance in between two discrete targets.² Nonetheless, even absent interpolation, many executive pay contracts contain some kind of discrete target. The Incentive Lab database covers the top 750 firms (measured by market capitalization) over 1998 to 2012, which encompasses 4673 unique CEO IDs. Of the 2,424 CEOs
that use absolute metrics, 1,666 (or 69%) have some discrete performance target. Of the 2,427 that use relative metrics, 1,088 or (45%) have some discrete performance target. To keep the analysis focused, I will examine only a single performance target with no interpolation. Of course, multiple performance targets would be a straightforward generalization of the theory developed here.

2.1 Risk Neutral Benchmark

A risk neutral principal (the firm) contracts with a risk neutral and effort averse manager (the agent). The manager exerts unobservable effort \( e \geq 0 \) at a cost of effort \( C(e) = 0.5ce^2 \), so \( C \) is strictly increasing and convex. The manager’s performance measure is given by

\[
q = e + \varepsilon,
\]

where \( \varepsilon \) follows the continuous unimodel and symmetric density \( g \) and distribution \( G \) with mean 0 and variance \( \sigma^2 \). Call \( e^* \) the first best effort, given as the solution to \( C'(e^*) = 1 \), so \( e^* = 1/c \). Observe that total surplus is \( e^* - C(e^*) = \frac{1}{2c} > 0 \). The firm offers the manager a contract \( (t, s, b) \), where \( t \) is the performance target, \( s \) is the salary, and \( b \) is the bonus, contingent on performance. The manager earns the bonus if performance exceeds the target:

\[
Pay = \begin{cases} 
  s & \text{if } q < t \\
  s + b & \text{if } q \geq t. 
\end{cases}
\]

This fits the simplest description of a performance target, where performance must exceed a threshold before the manager earns a payment. There is a discontinuity in the manager’s payoff, jumping from \( s \) to \( s + b \), when performance exceeds the target.

The probability that the manager receives his bonus is

\[
P \equiv \text{Prob}(q \geq t) = \text{Prob}(\varepsilon \geq t - e) = G(e - t),
\]

since by symmetry of \( g \), \( G(x) = 1 - G(-x) \). Observe that the probability increases in effort and decreases in the target:

\(^3\)The symmetry of the error distribution is not necessary, but does dramatically ease calculation. The most common distributions, such as normal, and fat-tailed distributions, like the Cauchy, are all symmetric.
Higher targets directly reduce the manager’s probability of achieving his bonus. Target and effort work in exactly opposite directions on this probability. The expected utility of the manager is

$$EU = s + bG(e - t) - C(e).$$  \(5\)

The manager can select effort at cost \(C(e)\) to maximize his expected payoff. The solution to this problem generates the incentive constraint for the manager:

$$bg(e - t) = C'(e).$$  \(IC\)

The manager equates the marginal cost of effort to the expected marginal benefit, which is the change in the probability of achieving the bonus, times the unconditional bonus itself. This marginal effect on the change in probability is represented by the term \(g\), and will permeate the analysis. While higher targets unilaterally decrease the probability of clearing the target, the effect on the change in probability is ultimately what matters. Indeed, the firm picks a contract which induces the manager’s effort, and the difference between the target and effort will ultimately drive the manager’s
incentives. As is common, assume the manager faces an outside opportunity \( \bar{u} \) in order to induce participation. The manager’s expected payoff must exceed this opportunity, and therefore impose the standard participation constraint \((PC)\) that \( EU \geq \bar{u} \).

The firm maximizes expected profits, subject to the incentive and participation constraints. The solution to this problem generates the optimal efficient contract. All proofs are in the appendix.

**Proposition 1** An optimal contract that implements first-best effort \( e^* \) is \((t^*, s^*, b^*)\) where

\[
t^* = e^*,
\]
\[
b^* = \frac{1}{g(0)},
\]
\[
s^* = \bar{u} + C(e^*) - \frac{1}{2g(0)}.
\]

The proposition provides one optimal contract, which in this case is efficient. This is not the unique optimal contract, though it is salient since \( t^* = e^* \). This should come as no surprise, as the manager is risk neutral, and there is no conflict of interest between the firm and the manager. Compare this to the usual efficient linear contract that makes the agent the full residual claimant on firm output, where the firm extracts rents from the manager through a (possibly) negative salary. Here, the efficient contract is nothing like the “sell the firm” contract. The firm will set the target to the efficient effort level, and then select the salary and bonus to solve the participation and incentive constraints, respectively. Proposition 1 formally proves that the firm can select the target equal to efficient effort under risk neutrality. This conforms to the common intuition that the target can equal the effort that the principal seeks to induce, which in this case is first-best effort.\(^4\)

The performance target offers a discrete jump in payoff if performance clears the target. Consider this a “prize” of \( b \), the difference in payoff from clearing the target versus not. In equilibrium, \( t^* = e^* \), so \((IC)\) in equilibrium becomes

\(^4\)Gutiérrez Arnaiz and Salas-Fumás (2008) offer a numerical example in which the performance standard is set equal to the mode of the performance distribution. But without formally solving for the optimal contract performance target, it is impossible to say for sure whether the target lies above or below equilibrium effort.
\[ b^* = \frac{1}{g(0)}. \]  

(9)

The relationship between target and effort is non-trivial, since a shift in the target \( t \) will immediately shift equilibrium effort \( e(s, b, t) \). However, the proof of Proposition 1 shows that because the manager’s participation constraint will bind, firm profits equal total surplus, and therefore the firm can afford to achieve efficiency. Given that the firm seeks to implement \( e^* \), the optimal target will pin down equilibrium efficient effort exactly. This occurs precisely when the returns to managerial effort are highest, the point when a marginal increase in effort leads to the greatest change in probability. This is exactly when the density \( g \) hits its maximum at \( t^* - e^* = 0 \).\(^5\) Moreover, the optimal salary compensates the manager for his outside opportunity and cost of effort, but then deducts half of his bonus from his salary upfront. Indeed, this is necessary in order to provide the manager with strong effort incentives. Comparative statics on the proposition immediately generate the following corollaries. First consider the effect from the changes in the outside options.

**Corollary 1** The optimal bonus is unchanged in the manager’s outside options \( \frac{\partial b^*}{\partial \bar{u}} = 0 \), while the optimal salary increases in the manager’s outside options \( \frac{\partial s^*}{\partial \bar{u}} > 0 \).

The participation constraint ensures that the manager meets his outside options. As with the canonical model, increasing outside opportunities forces the firm to increase the salary in order to retain the manager. Now consider changes in the cost of effort, which tracks the quality or productivity of the manager.

**Corollary 2** As the manager’s marginal cost of effort increases, the optimal target decreases, the optimal salary decreases, and the optimal bonus is unchanged.

Because our optimal target is \( t^* = e^* = \frac{1}{c} \), it is immediate that the firm will decrease the target as the manager’s cost of effort rises. In fact, this is the only comparative static in which the target changes. As effort becomes costly, it is efficient for the manager to work less, as that saves on manager disutility, and therefore social welfare. The corollary shows that even though the firm decreases the target, which decreases effort, it will simultaneously decrease salary. This is a countervailing effect to the fall in the target.\(^5\) The unimodel condition and symmetry imply that the mean of the distribution is its maximal point, which is why the firm will set the difference between target and effort to be equal to this mean of 0.
Indeed, the target is a powerful instrument and has a direct effect on effort. The salary counteracts this effect somewhat, even though effort will still fall in equilibrium.

There is a close theoretical analogy between rank-order tournaments (Lazear and Rosen 1981) and performance targets. Both rely on a relative comparison of output in order to secure an external prize. In performance targets, that comparison is against an exogenous standard set by the firm, whereas in tournaments that comparison is made against the output of another strategic player in the game. In both models, an increase in risk dampens incentives to provide effort, and both models can implement first best under risk neutrality. Here, the bonus reduces to $g(0)^{-1}$, which in the case of a normal distribution is simply $\sigma \sqrt{2\pi}$, so the bonus increases in risk unambiguously. This feature of how the bonus reacts to a change of risk is quite general, as I show next.

### 2.2 Increases in Risk

There’s hardly a more celebrated result in agency theory than the risk-incentives trade-off. The standard LEN model of linear contracts, exponential utility, and normal errors deviates from efficiency because of the risk premium that the firm must pay the manager to bear risk, through mean-variance preferences that include a disutility for risk. This workhorse model of contract theory, nicely summarized in Prendergast (1999), posits that as risk (measured through the variance of the error distribution) increases, optimal incentives should decrease, since the optimal bonus from that model is $(1 + r \sigma^2)^{-1}$. Because of this, the firm reduces the optimal bonus away from that which would guarantee efficiency.

The existing literature on the risk incentives trade-off has been mixed. Some papers find a positive relationship (Core and Guay 1999, Oyer and Shaefler 2001, Core and Guay 2002, Nam et al. 2003, and Coles et al. 2006), some find a negative relationship (Lambert and Larcker 1987, Aggarwal and Samwick 1999, and Jin 2002), and some find no relationship at all (Garen 1994, Yermack 1995, Bushman et al. 1996, Ittner et al 1997, and Conyon and Murphy 1999). Most of these papers measure risk as volatility of stock returns and measure incentives as pay-performance sensitivity, measured as changes in direct compensation for a given change in performance. The existing literature has not made a conclusive statement on whether incentives optimally increase or decrease with risk. This calls into question whether the theory is even valid, if it holds under such special circumstances. Indeed, a raft of papers have offered conditions under which the
Figure 2: An increase in risk. The steeper CDF second order stochastically dominates the flatter CDF.

trade-off reverses, giving a positive relationship between risk and incentives (e.g. Dutta 2008 and Prendergast 2002).

Here, the bonus is a reward to the manager for clearing the target, and as the incentive constraint shows, it will equilibrate the marginal cost of effort against the change in probability of clearing the target $g(e - t)$, times the unconditional “prize” of $b$. Unlike the canonical model, there is no disutility for risk that holds over the entire domain of the manager’s utility function. Rather, only incentives at the target matter. The assumption of risk neutrality here is to focus on a competing effect, namely the effect of noise on the probability of clearing the target.\textsuperscript{6} This effect will still permeate a model of risk aversion, though it may be muted because of the need for insurance. Of course, linear contracts allow no positive relationship for risk and incentives under any conditions.

\textsuperscript{6}Prendergast (2002) also assumes risk neutral agents in order to avoid the standard trade-off. It is possible that the trade-off may emerge under risk aversion, though the model does not permit closed forms solutions of this. Instead, numerical simulations do show that the risk incentives trade-off vanishes under risk aversion.
**Proposition 2**  
*As a variation in the manager’s performance measure increases, the optimal bonus increases.*

Said differently, as risk ($\sigma$) increases, this dampens the agent’s incentives to produce effort. To compensate for this reduction in incentives, the firm must increase the size of the prize, for the same logic as occurs in tournaments.\(^7\) Thus the optimal bonus exactly balances the increase in variance. This fits exactly Proposition 2b of Gutiérrez Arnaiz and Salas-Fumás (2008), who find the same unambiguous result that the bonus size increases in volatility. Even though Gutiérrez Arnaiz and Salas-Fumás (2008) use a more specific model (SVG process), they also find the same reversal of the risk-incentives trade-off.

The term $g(0)^{-1}$ is a proxy for the variance: As the variance on output rises, the tails of the density $g$ will increase and its maximal point $g(0)$ will sink. Recall that $G$ represents the probability of clearing the target, and $g$ is the change in this probability. So, under a higher variance, a marginal change in effort will lead to a smaller change in probability. It is the excess noise that forces the manager to reduce effort. Figure 2 shows two distribution functions, one that is second order stochastic dominant over the other. Remember that a marginal increase in effort changes the probability of achieving the target, and so it is the change in probability (the slope of the distribution function) that matters. In the low variance case, the slope of the distribution is steeper around the mean of 0, so a marginal increase in effort leads to a higher probability of hitting the target than under a high variance distribution. Proposition 2 proves this rigorously under two distributions.

There is a natural question of whether the bonus in this model can compare to the bonus in the linear model. Recall that the bonus in the linear model is the slope of the contract, and therefore directly maps into a conceptual definition of pay for performance. Because of the discrete nature of the target, there is no natural analogue to this slope, which is the marginal change in pay for a marginal unit of effort. Nonetheless, the matter is largely immaterial, because empirical estimates of PPS will almost always increase in the discrete bonus of this model.

For example, Aggarwal and Samwick (1999), Jin (2002), and Guay (1999) seek to estimate the risk-incentives trade-off by regressing pay for performance sensitivity (PPS)

\(^7\)A colorful analogy in tournaments (Lazear and Rosen 1981) is the play of tennis. Under normal conditions both parties exert effort. But imagine if the game occurs during a hurricane that vastly increases the variance on the error term. The ball can go anywhere, and this effectively dampens effort.
on risk, usually measured through the volatility of stock returns. Jin (2002) defines PPS as changes in total direct compensation, as well as changes in the re-evaluation of stock and stock options. An increase in the bonus of Figure 1 will increase total direct compensation, and therefore will have an upward effect on the empirical measure of PPS. This will confound the risk-incentives trade-off.

3 Risk Aversion

Now consider that the manager is risk averse and has a utility function $u$ that is strictly increasing and concave. The firm still writes a contract $(s, t, b)$ as before, with a similar bonus and target structure:

$$\text{Pay} = \begin{cases} 
  u(s) & \text{if } q < t \\
  u(s + b) & \text{if } q \geq t.
\end{cases}$$  \quad (10)

The discontinuity in the manager’s payoff now jumps from $u(s)$ to $u(s + b)$ when output exceeds the target. The expected utility of the manager is

$$EU = \int_{-\infty}^{t-e} u(s)g(\varepsilon)d\varepsilon + \int_{t-e}^{\infty} u(s + b)g(\varepsilon)d\varepsilon - C(e).$$  \quad (11)

The integral splits at $t-e$ because that is exactly the point for $\varepsilon$ such that the manager earns the bonus ($q \geq t$, or $\varepsilon \geq t - e$). Impose the standard participation constraint $(PC)$ that $EU \geq \bar{u}$. Observe that because the payoff to the manager takes only two discrete values $u(s)$ and $u(s+b)$, the firm can completely control the manager’s behavior through the choice of these two payoff levels. As such, the compensation terms pass out of the integral and we can re-write $EU$ as:

$$EU = u(s)G(t-e) + u(s+b)G(e-t) - C(e).$$  \quad (12)

Thus, the expectation makes the discrete payoff structure continuous, and so the manager can select effort to maximize his expected payoff. The solution to this problem, given by the first order condition, generates the incentive constraint for the manager:

$$g(t-e)\left(u(s+b) - u(s)\right) = C'(e).$$  \quad (IC')
As before, the benefit of effort includes its effect on changing the probability of clearing the target, expressed in the term \( g(e - t) \). Observe that under risk neutrality, the utility spread collapses to the bonus as a special case. The incentive constraint now contains the term \( u(s + b) - u(s) \), which I call the utility spread. This is the gain in utility from achieving the bonus. Since utility is increasing, the spread rises in the bonus \((u'(s + b) > 0)\). Immediate comparative statics on the incentive constraint generate the following results.

**Corollary 3** *Equilibrium effort rises in the bonus, falls in the salary, and rises in the target if effort exceeds the target \((\hat{e} > \hat{t})\).*

The first effect from bonus is the same as the canonical model: higher pay-for-performance sensitivity (PPS) induces the manager to work more. The third result is more surprising. In the LEN model, salary has no effect on effort incentives. Indeed, that is largely why the firm can hold the manager to his participation constraint, since it can lower salary without affecting \((IC)\). But now, salary affects the utility spread and therefore incentives. Under a general form of risk aversion, risk preferences at any point depend on wealth at that point. Specifically, because the manager has concave utility and therefore diminishing marginal utility, for a fixed bonus, a higher salary will cause the utility spread to shrink (since \(u'(s + b) < u'(s)\)). This will decrease effort incentives.

The first best solution maximizes total surplus, the profits of the firm plus the utility of the manager. Because the manager is risk averse, the compensation terms do not fall out of the surplus function, as they did in the risk neutrality case. Now, the social planner maximize surplus subject to the participation constraint. In the bonus and
target setting, we arrive at the usual benchmark: if the firm can contract on effort, it will provide full insurance to the agent:

**Corollary 4** Under risk aversion, the first best gives full insurance of the manager, with a flat salary given by \( u'(s^*) = \frac{1}{\lambda} \), where \( \lambda \) is the multiplier on the participation constraint.

The flat salary results from optimal risk-sharing, since the firm is risk neutral and the manager is risk averse. This is the Borch condition that makes equal the marginal utility of the principal and agent. Since the principal is risk neutral, his marginal utility is 1. The term \( \lambda \) in the solution is the multiplier on \((PC)\), and \( \lambda > 0 \) since \((PC)\) binds in equilibrium.

The concavity of the utility function prevents first best and will lead to an effort distortion. To see this, imagine that the firm could implement first best effort. Plugging this into the incentive constraint generates \( g(e - t) \Delta = 1 \), where \( \Delta \) is the utility spread. Rearranging terms gives

\[
g(e - t) = \frac{1}{\Delta} > \frac{1}{b^*} = g(0)
\]

where the bonus on the right hand side is set at the first best level, and \( \Delta < b^* \) by risk aversion. But of course, this is impossible since the distribution peaks at zero. So, in fact, \((IC)\) will hold at an effort level distorted away from first best. This occurs precisely when the utility spread is smaller than the optimal bonus, which must occur since the manager is risk averse and the optimal target lies away from the efficient effort \((t < e^*)\). Indeed, both the utility spread falls short of the bonus, and the change in probability lies beneath its maximal point. And thus the marginal benefit is less than the first best marginal cost of one, yielding the effort distortion.

The firm maximizes expected profits, subject to the incentive and participation constraints. The full program involves expected profits less a multiplier for both constraints:

\[
\max_{(s,b,t)} e - (s + bG(e - t))
\]

subject to

\[
u(s) + \left( u(s + b) - u(s) \right) G(e - t) - C(e) \geq \bar{u}, \quad \text{\((PC)\)}
\]

\[
g(e - t) \left( u(s + b) - u(s) \right) = C'(e). \quad \text{\((IC)\)}
\]
Proposition 3 solves for this program and we discover that the target plays an important role in balancing the risk and incentives problem:

**Proposition 3** Under risk aversion, the incentive and participation constraints both bind. The optimal target lies below equilibrium effort ($\hat{t} < \hat{e}$).

The primary result is that the firm will shade the target downward to handle the manager's risk aversion. Proposition 3 shows that the target, in addition to the bonus, also offers insurance. This removes the insurance burden from the bonus and onto the target, as often occurs when the firm has multiple instruments to design optimal compensation. Recall under the benchmark model that the incentive constraint equalizes the marginal cost of effort against its marginal return. In the structure of this model, that is equivalent to a horizontal line passing through the distribution of effort, as shown in Figure 3.

This occurs because of the symmetry of the error distribution. From $(IC)$, the marginal cost of effort must equal the marginal return, which is the marginal change in the probability of clearing the target times the size of the prize, the utility spread. Because $g$ is symmetric, there will always be two targets symmetrically distributed around equilibrium effort that solve $(IC)$. To see this visually, imagine a horizontal line passing through the density $g$. The coordinates of the $x$-axes of the intersection points are the optimum targets that satisfy $(IC)$. There will always be two solutions to this problem, as Figure 3 illustrates.

The low and high targets will equivalently induce the same equilibrium effort. Recall that the probability of clearing the target, $P$, decreases in the level of the target; as such, the manager has a lower chance of receiving the bonus with high targets. Therefore, the manager receives a higher expected bonus from a low target rather than a high target, so he requires less salary in order to participate. Said differently, the principal must pay a premium to the manager in order to induce participation under a high target. Since both targets generate the same equilibrium effect, the high target has no benefit for output, only a higher cost to induce participation.

This result follows fundamentally from risk aversion. Recall that the utility spread is the difference in utility from receiving the bonus versus just receiving the salary alone. For any fixed bonus, this spread falls in the salary level because of diminishing marginal utility (driven by the risk aversion, illustrated in Figure 3(b)). Therefore, when the firm offers a high target with a low probability of payout, it must offer a corresponding
high salary to guarantee participation. That high salary, call it $s_H$, paired with a given bonus, call it $b_H$, determines the utility spread and therefore effort incentives. A low target raises the probability of payout, and the firm can afford to pay a lower salary to guarantee participation. Because of diminishing marginal utility for a fixed bonus $b_H$, the utility spread at the low salary will exceed the utility spread at the high salary, since the utility curve is steeper at the lower salary level. To keep incentives unchanged, the firm can therefore lower the bonus to some $b_L < b_H$, which will match exactly the utility spread and therefore the incentives at the prior contract. To see this visually, observe in Figure 3(b) that the diminishing marginal utility (risk aversion) forces $b_L < b_H$ in order for incentives ($\Delta$) to be identical at both contracts. Thus, the low target pairs with a low salary and low bonus, and offers the same incentives as the high target with a high salary and high bonus, inducing identical effort at lower cost.

As such, in every equilibrium we always have $\hat{t} < \hat{e}$. The full solution in the proof of Proposition 3 gives the optimal contract as a function of the constraints of the problem, namely, the two Lagrangian constraints on the incentive and participation constraints. The program is inherently complex as salary, bonus and target jointly and simultaneously determine effort. It is impossible to change one variable alone without changing others as well. Thus, the LEN logic, where a change in salary will not affect the incentives, no longer holds. Still, even with this complexity, I show that the participation constraint will always bind. The firm will always select the target such that it extracts the full rent out of the manager.

To see why, observe that the firm has at its disposal a target that can always serve as an extra instrument to modulate the manager’s expected utility. If the target is strictly below equilibrium effort, so $\hat{t} < \hat{e}$, then a slack participation constraint leaves rents for the manager. But the firm can always simultaneously lower the salary and raise the target, keeping equilibrium effort constant. Lowering the salary will tighten the participation constraint, since the manager’s expected payoff will fall. Raising the target will further tighten the participation constraint, as the manager certainly prefers low targets to high targets. But both these actions will raise effort and, therefore, profits for the firm, and thus allow the firm to implement the same effort at a lower cost (a lower salary and lower expected compensation). The firm will do this until the participation constraint binds.

Figure 4 shows numerical simulations from an example with quadratic utility and normal errors. In the top graphs, I show the optimal contract as a function of the outside
Figure 4: Numerical simulations under quadratic utility and mean-zero normal errors. The top graphs show the optimal contract as a function of the outside option, when variance is fixed at 1. The bottom graphs show the optimal contract as a function of variance, when the outside option is fixed at zero.

options, holding the variance to 1. In the bottom graph, I show the optimal contract as a function of the variance, setting the outside option to zero. Notice that in both graphs, the optimal target lies below equilibrium effort. The optimal bonus is $b = 10$, suggesting that the insurance effect that dampens the optimal bonus exactly balances the effect from Proposition 2. Thus, in this case with quadratic utility, there is indeed no risk incentives trade-off, as the optimal bonus is constant as a function of risk.
Figure 5: Timeline of the private information model.

4 Managerial Ability

I now extend the model to include private information. To ease analysis, assume the manager is risk neutral, as this will focus on the effects of private information on the optimal contract. The firm’s output depends on managerial expertise given by $\theta$, which captures the productivity, ability or skill of the manager. The manager knows $\theta$, but the firm only knows that it follows a density $f$, with distribution $F$, over the interval $[\hat{\theta}, \bar{\theta}]$. The manager has private, pre-contract information about $\theta$. Figure 5 shows the timeline of the private information game. I impose the standard regularity condition that $F(\theta)/f(\theta)$ is increasing in $\theta$, and the inverse hazard rate, $(1 - F(\theta))/f(\theta)$, is decreasing in $\theta$. These monotonicity conditions are common in the private information agency literature.\footnote{Bagnoli and Bergstrom (2005) shows that these conditions are satisfied by many common distributions, such as: uniform, normal, exponential, gamma, and others.}

As before, the manager exerts costly and unobservable effort, and his output is stochastic:

$$q = \gamma \theta + e + \varepsilon. \quad (15)$$

The parameter $\gamma \geq 0$ tracks the marginal productivity of the manager’s ability. Higher $\gamma$ managers are inherently more productive. The error term $\varepsilon$ follows the same assumptions as in the earlier sections, such as symmetry and the regularity condition. As before, the manager enjoys the outside option $\bar{u}$, which we normalize here to zero. The manager will accept the contract if his expected payoff exceed his outside option, given by the participation constraint.

I allow communication between the manager and the firm, based on messages depending on the manager’s private information. The firm will therefore design a compensation schedule as a function of these messages. By the revelation principle, I restrict attention
to contracts that rely on a report $\hat{\theta}$ about ability $\theta$ (a standard assumption in the private information contracting literature). The contract has the same structure as before: the manager earns a salary, and receives a bonus if output clears the target. The firm picks a contract $(s(\hat{\theta}), b(\hat{\theta}), t(\hat{\theta}))$, which depends on the manager’s report $\hat{\theta}$. Formally, the manager’s pay is given by:

$$\text{Pay}(\hat{\theta}, q) = \begin{cases} 
  s(\hat{\theta}) & \text{if } q < t(\hat{\theta}) \\
  s(\hat{\theta}) + b(\hat{\theta}) & \text{if } q \geq t(\hat{\theta}). 
\end{cases} \quad (16)$$

The probability that the manager receives his bonus is:

$$P = \text{Prob}(q > t(\hat{\theta})) = G \left( \gamma\theta + e - t(\hat{\theta}) \right), \quad (17)$$

by the symmetry of $g$. Note that this probability is the function both of his true ability $\theta$ as well as his report $\hat{\theta}$. This occurs because the target is a now function of $\hat{\theta}$. The expected utility of the manager when his true utility is $\theta$ but he reports $\hat{\theta}$ to the firm is:

$$EU(\hat{\theta}, \theta) = s(\hat{\theta}) + b(\hat{\theta})G \left( \gamma\theta + e - t(\hat{\theta}) \right) - C(e). \quad (18)$$

Once again, this is both the function of his report $\hat{\theta}$, which affects his contract, as well as his true type $\theta$, which affects his probability of success. In addition, the firm must induce the manager to report truthfully, so the firm must pick a contract that such that the manager’s expected payoff reporting the true $\theta$ exceeds the payoff any other report $\hat{\theta}$. Thus, impose the incentive compatibility constraint, given by $EU(\theta) \equiv EU(\hat{\theta}, \theta) \geq EU(\theta, \theta)$ for all $\theta, \hat{\theta}$. The term $EU(\theta)$ is the manager’s expected utility when he truthfully reports his type $\hat{\theta} = \theta$. Taking the first derivative of his expected utility with respect to effort generates:

$$b(\theta)g \left( \gamma\theta + e - t(\theta) \right) = C'(e). \quad (EIC)$$

Call this the effort incentive constraint (EIC), in contrast to his incentive compatibility constraint (IC). As before, higher bonuses induce more effort from the manager, and (EIC) balances the marginal cost of effort against its marginal benefit, which is the bonus times the change in probability of hitting the target. The firm will choose the contract to maximize expected profits net of compensation to the manager, subject to the
effort incentive constraint, the incentive compatibility constraint, and the participation constraint. Therefore, the firm chooses the contract to solve the program:

\[
\max_{\langle e(\theta), s(\theta), b(\theta), t(\theta) \rangle} \int_{\theta}^{\hat{\theta}} \left[ \gamma \theta + e(\theta) - s(\theta) - b(\theta) G \left( \gamma \theta + e(\theta) - t(\theta) \right) \right] f(\theta) d\theta
\]

subject to:

\[
EU(\theta) \geq EU(\hat{\theta}, \theta) \text{ for all } \theta, \hat{\theta}, \quad \text{(IC)}
\]

\[
EU(\theta) \geq 0 \text{ for all } \theta, \quad \text{(PC)}
\]

\[
\frac{b(\theta)}{e} g \left( \gamma \theta + e(\theta) - t(\theta) \right) = e(\theta) \text{ for all } \theta. \quad \text{(EIC)}
\]

I write the effort as a function of \( \theta \) because the contract induces a specific effort level for each \( \theta \).

Observe that if \( \theta \) is known to both parties and contractible, then the firm’s problem simplifies to the symmetric information setting. The incentive compatibility constraints fall away, and the firm can observe \( \theta \) and directly contract on it. The firm no longer worries about truth-telling but rather simply solves the optimal effort problem which was solved in Section 2. The optimal contract that induces a first best effort level will be similar to the efficient contract in the benchmark model without managerial ability.

To gain traction on the firm’s optimization problem in (19), a key analytical simplification is to consider the marginal benefit function \( MB(\theta) \), which is the left hand side of the manager’s effort incentive constraint:

\[
MB(\theta) = b(\theta) g \left( \gamma \theta + e - t(\theta) \right) . \quad \text{(20)}
\]

In equilibrium, the firm will select the contract to balance the marginal cost of effort against its marginal benefit, which is the bonus \( b(\theta) \) times the change in probability of hitting the target \( g \left( \gamma \theta + e - t(\theta) \right) \). Under quadratic cost of effort, this reduces to \( e(\theta) = \frac{MB(\theta)}{c} \). Therefore, the marginal benefit function directly will determine the optimal effort induced for any given level of managerial ability \( \theta \).

As shown in the appendix, the incentive compatibility constraints and the participation constraint together imply that the manager’s expected utility takes the following
form:

\[ EU(\theta) = \int_{\theta}^{\theta} \gamma MB(x)dx. \]  

(21)

So long as the bonus function is non-zero, this equation shows that the manager will earn informational rents, since his expected utility \( EU(\theta) \) will be non-negative. Moreover, these informational rents will increase for high levels of managerial ability.

To see why, observe that because his expected bonus \( b(\theta)G(\gamma \theta + e - t(\theta)) \) increases in his ability, in order to guarantee participation the firm can afford to offer a lower salary to a high type manager. If \( \theta \) was known and contractible, then the high type manager would be tempted to understate his ability in order to avoid this decrease in salary. To compensate for this, the firm must therefore provide informational rents to the high type managers, and indeed these rents will increase in the level of managerial ability. This prevents the high types from mimicking the low types (i.e. preventing the highly able managers from announcing that they are actually lower quality in order to avoid receiving the lower fixed salary). This result leads to our next proposition, which gives the properties of the optimal contract under private information.

**Proposition 4** There exists an equilibrium where (1) the optimal bonus \( b(\theta) \) and target \( t(\theta) \) increase in \( \theta \), and (2) the optimal contract is given by:

\[
    b^*(\theta) = \max \left\{ 0, \frac{1 - \gamma \frac{1-F(\theta)}{f(\theta)}}{g(\gamma \theta + e(\theta) - t^*(\theta))} \right\}.
\]

(22)

The proposition shows the existence of an equilibrium in which the bonus and target are both increasing functions of ability. Figure 6 shows the equilibrium marginal benefit function, and equilibrium target. The marginal benefit function is monotonic in \( \theta \) while the optimal target will exceed \( \gamma \theta + e(\theta) \), the sum of two increasing functions. This provides conditions under which the optimal contract increases in ability. Observe that the optimal contract specifies a joint condition on both the equilibrium bonus and target simultaneously. Indeed, the equilibrium essentially determines the optimal marginal benefit condition given by \( MB(\theta) = 1 - \gamma H(\theta) \), where \( H(\theta) \) is the inverse hazard rate \( \frac{1-F(\theta)}{f(\theta)} \). The proof replaces the general (IC) constraints with a local incentive compatibility and monotonicity condition, and then shows that the solution to this program yields the equation above.

Figure 6 shows the marginal benefit function \( MB(\theta) \) is increasing over its support \([\underline{\theta}, \bar{\theta}]\). Observe that at the highest type \( \bar{\theta} \), we have \( MB(\bar{\theta}) = 1 - \gamma H(\bar{\theta}) = 1 \). Recall that
in the benchmark model without managerial types, the marginal cost of the manager was always $C'(e^*) = 1$. Therefore, the optimal contract guarantees that the marginal benefit equals the first best level of marginal cost for $\bar{\theta}$. Said differently, the manager of type $\theta$ receives an informational rent in order to induce him not to misreport his ability. This informational rent is highest for the highest type. The marginal benefit at that point is so high that it can equal the marginal cost at the efficient frontier. For everyone else, there is a distortion.

The shape of the bonus and target functions, $b(\theta)$ and $t(\theta)$, directly follow from the monotonic nature of the marginal benefit function. The proof of Proposition 4 provides the details, but the intuition is straightforward. Every manager of type $\theta$ is tempted to underreport his ability in order to avoid the lower salary. In response, the firm pays a higher level of incentive compensation. The highly able managers will be tempted not to misreport, because they now receive more of their total compensation in variable pay. This variable pay takes the form of a higher bonus and a harder target, since a high bonus and high target increase incentives of the contract. This is why the optimal bonus and target will both increase in managerial ability.

Unlike the model of risk aversion in which there is a unique target selected in equilibrium, this model of managerial ability and risk neutrality involves two targets that can both simultaneously exist. The low and high target provides the firm two options:
If the firm offers a low target, the manager’s expected compensation is high because his probability of clearing that target is high, compared to a high target. As such, the firm can afford to attract the manager with a lower salary if it offers a lower target. The firm will therefore pick the salaries for the low and high target, respectively, such that the participation constraint binds, making the agent indifferent between the two packages \((s_L, b, t_L)\) and \((s_H, b, t_H)\). Both induce the same effort and hold the manager to his reservation utility. Because the firm is risk neutral, the salary does not affect incentive (unlike \((IC)\) under risk aversion). So the two packages have no effect on firm profitability, making the firm also indifferent between these two targets. The equilibrium in the proposition outlines the case with the high target, where it is straightforward to show that the marginal benefit function and the optimal target increase in ability.

4.1 Information Risk

Suppose the ability distribution \(f\) is distributed uniformly over support \([-a, a]\). Here, the parameter \(a > 0\) is the support of the distribution and therefore tracks the variance \(a^2/3\) of the distribution, and is therefore a proxy for the level of uncertainty in the system. Plugging in the uniform distribution into the optimal contract generates the optimal bonus of \(b^*(\theta) = (1 - \gamma H(\theta))2a\). There are a few noteworthy features of this contract.\(^9\)

First, observe that the optimal bonus increases in \(\theta\) because the inverse hazard rate \(H(\theta) = (1 - F(\theta))/f(\theta)\) is decreasing in \(\theta\), consistent with Proposition 4. Second, the optimal bonus increases in \(a\), and therefore increases in uncertainty. This provides a closed form solution for how the firm will raise the bonus after an increase in risk. As before, higher risk dampens effort incentives, and the firm raises the bonus in order to counteract this effect. Finally, observe that the optimal effort function simplifies to \(e(\theta) = \frac{1 - \gamma H(\theta)}{c}\), which also is increasing in managerial ability.

Baker and Jorgensen (2003) and Dutta (2008) have called the variation on ability “information risk,” and they each provide different conditions under which incentives increase in this risk. For example, Dutta (2008) assumes that the manager enjoys type-

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\(^9\)Note that the uniform distribution is not strictly unimodal, since it is flat everywhere. Because of this, it is impossible to pin down an optimal target since the curvature of the density is necessary to do so. Indeed, under a uniform distribution, the change in probability is constant for any choice of effort and target, so the firm is indifferent between all possible targets. Nonetheless, it does provide a closed-form solution for the bonus that is tractable and straightforward.
contingent outside options that increase in the manager’s ability, and finds that if these outside options are sufficiently general (rather than firm-specific), then there will be a positive relationship between incentives and information risk. Moreover, Rajan and Saouma (2006) gives conditions for a negative relationship between incentives and information risk, though their framework is a subset of the model in Dutta (2008). My finding here that information risk and incentives move together is broadly consistent with these papers.

One empirical measure of variation in ability may be the industry or age of the manager. For example, it is conceivable that the variation in ability is large for younger managers, as well as in new economy knowledge-intensive industries, where the market has not learned the manager’s type in both settings. Contrast this to older managers and more traditional industries (like manufacturing), where the ability distribution may be more narrow. These could provide empirical tests of the positive relationship between information risk and incentives.

5 Conclusion

Until very recently, academic researchers have largely guessed what executive contracts actually look like. In the face of such lack of knowledge about these specific contracts, linear models are good first approximations, given their simplicity and robustness. This has generated the large LEN literature in accounting and finance (on the theory side) coupled with linear tests of the risk incentives trade-off (on the empirical side). Yet, the empirical tests of the risk incentives trade-off remain weak, and a comprehensive test that combines models of actual contracts, with a precise fit to empirical data remains elusive.

But since the SEC required more disclosure of executive contracts in 2006, we now have a better sense of what form CEO pay actually takes. A noteworthy feature of these contracts is the reliance on a performance target of some kind, which involves an explicit payout according to a pre-specified target. This paper models such contracts explicitly and generates a host of new intuitions and insights that can be tested against executive pay data: (1) The optimal bonus increases in risk, (2) the target provides insurance (in addition to the bonus) to help resolve the managers moral hazard problem, and (3) the optimal bonus and target increase in the ability of the manager.

The field is ripe for further explanation and exploration of contracts that firms ac-
tually use. The trend towards more disclosure makes these contracts available to the analyst, who can then tailor the theory and generate more precise empirical predictions than were possible before. This new research agenda mixes theory with empirics at a more intimate level, since the contract itself emerges from practice. I remain optimistic for how future work can explore dynamic effects, earnings management, the informativeness principle, team incentives, and many of other theoretical questions of contracts used in practice, of which performance targets are just one example.

6 Appendix

Proof of Proposition 1: Consider a contract \((t, s, b)\). Maximizing expected payoff to the manager with respect to effort gives the incentive constraint (IC). Assume the second order condition holds:

\[-g'(t - e^*)b - C''(e^*) < 0.\] (SOC)

Because both parties are risk neutral and the firm is the contract designer, the firm’s expected profit will equal expected total surplus. Therefore, it is possible to implement first best. To do so, first note that \(C'(e^*) = 1\). We seek to implement first best effort \(e^*\), which satisfies \(C'(e^*) = 1\). Plug in \(t^* = e^*\) and \(e = e^*\) into (IC), and call this the equilibrium incentive constraint:

\[b^* = \frac{1}{g(0)}.\] (23)

The participation constraint must hold:

\[EU = s + bG(e - t) - C(e) \geq \bar{u}.\] (24)

Now \(G(e^* - t^*) = G(0) = \frac{1}{2}\), so this becomes

\[\frac{1}{2}(2s + b) \geq \bar{u} + C(e^*).\] (25)

Lowering the salary by a small amount will maintain the participation constraint but not affect the incentive constraint, and therefore increase firm profits. So, (PC) binds. Plugging in (23) above into (PC) gives

\[s^* = \bar{u} + C(e^*) - \frac{1}{2g(0)}.\] (26)

\(^{10}\)An open question is the firms desire to discourage earnings management. A full exploration of this phenomenon is outside the scope of this paper but worthy of future research. Bizjak et al. (2014) finds empirically that performance targets induce real earnings management rather than accruals management, but to date there is no theoretical investigation of this question.
Therefore the contract \((t^*, s^*, b^*)\) implements \(e^*\) and satisfies \((SOC)\), \((IC)\) and \((PC)\).

**Proof of Corollary 3:** For shorthand, denote

\[
\Delta := u(s + b) - u(s) > 0 \text{ and } \Delta' := u'(s + b) - u'(s) < 0 \tag{27}
\]

because utility is increasing concave. Recall that the incentive constraint is

\[
g(e - t)[u(s + b) - u(s)] = C'(e) = ce. \tag{(IC)}
\]

Write the second order sufficient condition from the manager’s effort problem as

\[
g'(e - t)\Delta - c < 0. \tag{(SOC)}
\]

Differentiate \((IC)\) with respect to the bonus:

\[
\frac{\partial e}{\partial b} = \frac{g(e - t)u'(s + b)}{c - \Delta g'(e - t)} > 0. \tag{28}
\]

This occurs because \(u\) is increasing and from \((SOC)\). Now, differentiate \((IC)\) with respect to salary:

\[
\frac{\partial e}{\partial s} = \frac{g(e - t)\Delta'}{c - \Delta g'(e - t)} < 0, \tag{29}
\]

from \((SOC)\). Finally, differentiate \((IC)\) with respect to the target:

\[
\frac{\partial e}{\partial t} = -\frac{g'(e - t)}{c} \geq 0 \text{ iff } t \leq e, \tag{30}
\]

because \(g\) is increasing over its positive domain and decreasing over its negative domain. From the proof of Proposition 3, we know \(\hat{t} < \hat{e}\) in equilibrium and therefore the derivative above is always positive in equilibrium.

**Proof of Corollary 4:** Suppose the firm can contract directly on effort. Then there is no incentive constraint but only a participation constraint. The firm maximizes profit subject to participation, or solves:

\[
\max_{(s,b,t)} e - (s + Gb) + \lambda(u(s) + \Delta G). \tag{31}
\]

Differentiating with respect to salary gives

\[
\lambda u'(s) + \Delta' G = 1. \tag{32}
\]

Differentiating with respect to the bonus gives
\[ u'(s + b) = \frac{1}{\lambda}. \]  

(33)

Combining these two equations shows

\[ u'(s) = u'(s + b) = \frac{1}{\lambda}. \]  

(34)

Because the utility function is strictly monotonic, this means \( b = 0 \). Total surplus of the firm is

\[ e - (s + Gb) + u(s) + \Delta G - C(e). \]  

(35)

Maximizing with respect to effort gives the first best level \( e^* = \frac{1}{\epsilon} \). Therefore we can implement first best with the following contract: \( t^* = e^* = \frac{1}{\epsilon}, b^* = 0, u'(s^*) = \frac{1}{\lambda} \).

\[ \blacksquare \]

Proof of Proposition 3: For short hand, we use the following notation to ease exposition:

\[ g = g(e - t) = g(t - e), \frac{\partial g}{\partial e} = g' = -\frac{g}{g'}, G = G(e - t), G(t - e) = 1 - G. \]  

(36)

The firm maximizes profits subject to (IC) and (PC) given by the program in \( x \). Write the Lagrangian as

\[ L = e - (s + bG) + \lambda(u(s) + \Delta G) + \mu(g\Delta - C'(e)). \]  

(37)

Recall that the chain rule gives

\[ \frac{\partial \pi}{\partial k} = \Pi(e, k) \frac{\partial e}{\partial k} + \Pi_k(e, k) = 0. \]  

(38)

Therefore we have the following partial derivatives:

\[
\begin{align*}
L_e &= 1 - bg + \lambda(\Delta g) + \mu(g'^2 - C\epsilon) \\
L_b &= -G + \lambda(u'(s + b)G + \mu gu'(s + b) \\
L_s &= -1 + \lambda(u'(s) + (u'(s + b) - u'(s))G) + \mu g(u'(s + b) - u'(s)) \\
L_t &= bg + \lambda(\Delta(-g)) + \mu(\Delta(-g')).
\end{align*}
\]  

(39-42)

Differentiate \( L \) with respect to \( b \) to get

\[ [(1 - bg) + \lambda \Delta g + \mu(g' \Delta - c)] \frac{gu'(s + b)}{c - \Delta g'} - G + \lambda u'(s + b)G + \mu gu'(s + b) = 0. \]  

(43)

Observe that \( \mu(g' \Delta - c) \) cancels out by (IC):

\[ (1 - bg)gu'(s + b) + \lambda[\Delta g^2 u'(s + b) + u'(s + b)G(c - \Delta g')] = G(c - \Delta g'). \]  

(44)

Rearrange terms to arrive at

\[ \frac{1}{u'(s + b)} = \lambda + \frac{(1 - bg)g + \lambda \Delta g^2}{GX}. \]  

(45)
where $X = c - \Delta g' > 0$ by (SOC). Differentiate with respect to salary to get
\[
[(1 - bg) + \lambda \Delta g + \mu (g' \Delta - c)] \frac{g \Delta}{c - \Delta g'} - 1 + \lambda [u(s) + \Delta' G] + \mu g \Delta' = 0. \tag{46}
\]
Once again the terms $\mu (g' \Delta - c)$ cancel out, so
\[
(1 - bg) + \lambda \frac{\Delta g^2}{c - \Delta g'} \Delta + u'(s) + \Delta' G = 1. \tag{47}
\]
Differentiate $L$ with respect to $t$ to get
\[
[(1 - bg) + \lambda \Delta g + \mu (g' \Delta - c)] \left( -g \frac{\partial e}{\partial t} \right) + bg - \lambda \Delta g - \mu \Delta g'. \tag{48}
\]
By the standard results, equations (45), (47), (48) are necessary and sufficient for an optimal solution.
The second order condition from the incentive constraint satisfies
\[
-g'(t - e)(u(s + b) - u(s)) - C''(e) < 0. \tag{SOC}
\]
Throughout, use the shorthand $\Delta = u(s + b) - u(s)$. Because costs are quadratic, we can rewrite (SOC) as
\[
c + g'(t - e)\Delta > 0. \tag{49}
\]
From Corollary 3, (SOC), and (30) we have:
\[
\frac{\partial e}{\partial t} - 1 = \frac{-c}{c + g'(t - e)\Delta} < 0. \tag{50}
\]
Differentiate expected utility with respect to the target:
\[
u(s) g(t - e) \left( 1 - \frac{\partial e}{\partial t} \right) + u(s + b) g(e - t) \left( \frac{\partial e}{\partial t} - 1 \right). \tag{51}
\]
Inserting (50) and rearranging gives,
\[
\frac{\partial EU}{\partial t} = g(t - e) \left( \frac{\partial e}{\partial t} - 1 \right) \Delta < 0, \tag{52}
\]
where we have used (SOC) from (IC). Therefore, raising the target lowers the manager’s payoff.
Now differentiate expected utility with respect to the salary:
\[
u'(s) g(t - e) \left[ -\frac{\partial e}{\partial s} \right] + u'(s + b) g(e - t) + \left( \frac{\partial e}{\partial s} \right) - C'(e) \frac{\partial e}{\partial s}. \tag{53}
\]
From (SOC) and (29), we have,
\[
\frac{\partial EU}{\partial s} = \frac{\partial e}{\partial s} \left[ g(e - t) \Delta' - C'(e) \right] > 0. \tag{54}
\]
Therefore, lowering the salary lowers expected utility of the manager. The function $e(s, b, t)$ is given by the manager’s incentive constraint, which determines his effort with respect to $(s, b, t)$. We now wish to show that $\hat{e} < e(\hat{s}, \hat{b}, \hat{t})$. 29
Suppose \((s, b, t)\) is an optimal contract. We know the firm cannot implement first best, so \(t \neq e\). First suppose \(t = t_L = e - z\) for some \(z > 0\). By Lemma 2, there exists a \(t_H\) that also satisfies the incentive constraint and implements the same effort level:

\[
C'(e) = g(e - t_L)\Delta = g(e - t_H)\Delta, \quad (55)
\]

where \(t_H = e + z > e - z = t_L\). But this higher target lowers the manager’s expected pay given \((s, b)\):

\[
EU(s, b, t_H) = u(s) + \Delta G(e - t_H) < u(s) + \Delta G(e - t_L) = EU(s, b, t_L) = \bar{u} \quad (56)
\]

where the last equality holds since (PC) binds by Lemma 1. Thus \((s, b, t_H)\) violates (PC) and cannot be optimal. Now suppose \(t = t_H = e + z\) for some \(z > 0\), for the given optimal contract \((s, b, t)\), which we can call \((s_H, b_H, t_H)\). Again by Lemma 2, there exists a \(t_L\) that implements the same effort. By the same logic, this low target raises the manager’s expected pay, thus relaxing (PC). But then the firm could simultaneously lower the salary to \(s_L < s_H\) and bonus to \(b_L < b_H\) such that (PC) tightens and incentives are unchanged:

\[
\Delta(s_L, b_L) = u(s_L + b_L) - u(s_L) = u(s_H + b_H) - u(s_H) = \Delta(s_H, b_H) \quad (57)
\]

This implements effort with the lower target at lower cost. Therefore, \((s_H, b_H, t_H)\) cannot be optimal. Thus \(\hat{t} < \hat{e}\).

**Proof of Proposition 2:** I use second order stochastic dominance to measure an increase in the dispersion of the distribution. Assume \(g_i\) for \(i = 1, 2\) are two probability densities over the real line with mean 0 and finite variance that both satisfy the single-peaked condition. Suppose \(G_1\) is second order stochastic dominant over \(G_2\). By definition, for all \(w \in (-\infty, \infty)\), we have \(S_1(w) < S_2(w)\) and \(S_1(\infty) = S_2(\infty)\) where

\[
S_i(w) = \int_{-\infty}^{w} G_i(w)dw. \quad (58)
\]

Suppose \(g_2(0) > g_1(0)\). By the single-peaked condition, \(g\) is increasing over its negative domain, so \(g_i(0) > g_i(x)\) for each \(x < 0\). Now \(g_2(0) > g_1(0)\), both densities are strictly increasing and they both integrate to the same value, \(G_1(0) = G_2(0) = \frac{1}{2}\), at the end point of the interval \((-\infty, 0)\). Then there exists a \(z \in (-\infty, \infty)\) such that

\[
g_2(x) < g_1(x), \ \forall x < z. \quad (59)
\]

Integrate both sides of this inequality over \((-\infty, x)\) for each \(x < z\) to generate

\[
G_2(x) < G_1(x), \ \forall x < z. \quad (60)
\]

Integrate over \((-\infty, z)\) to arrive at
This contradicts the definition of SOSD. Therefore $g_2(0) < g_1(0)$, and so the optimal bonus from Proposition 1 is

$$b_1^* = \frac{1}{g_1(0)} < \frac{1}{g_2(0)} = b_2^*. \tag{62}$$

\[\blacksquare\]

**Lemma 1** If $t \leq e(\hat{s}, \hat{b}, \hat{t})$ for an optimal contract $(\hat{s}, \hat{b}, \hat{t})$, then (PC) binds.

**Proof of Lemma 1:** Suppose (PC) is slack. Consider $\hat{t} < e(\hat{s}, \hat{b}, \hat{t})$. Then the firm can raise the target by a small amount, reducing expected utility and maintaining (PC) because it is slack. This raises effort because effort exceeds the target and $g$ is increasing over its negative domain ($\frac{\partial g}{\partial t} > 0$ by (30) since $t < e(s, b, t)$). So effort rises, as does expected output and the firm’s expected profit. This contradicts that $(\hat{s}, \hat{b}, \hat{t})$ was optimal.

Suppose $\hat{t} = e(\hat{s}, \hat{b}, \hat{t})$. Consider some salary $s' < \hat{s}$. This tightens (PC) by (54) and raises effort by (29) so $e(s', \hat{b}, \hat{t}) > e(\hat{s}, \hat{b}, \hat{t}) = \hat{t}$. Raising the target will raise effort by (30), so increase $\hat{t}$ to $t'$ such that $e(s', \hat{b}, t') = t'$.

This is possible since $\frac{\partial t}{\partial t} < 1$ by (50). This further tightens (PC) by (50) and raises profits since output is higher (from effort), salary is lower ($s' < \hat{s}$) and $P$ is lower ($\frac{\partial P}{\partial t} < 0$). This contradicts that $(\hat{s}, \hat{b}, \hat{t})$ optimal.

\[\blacksquare\]

**Lemma 2** If $t = e - z$ implements equilibrium effort for some $z$, then so does $t = e + z$.

**Proof of Lemma 2**\textsuperscript{11}: Suppose $(s(\theta), b(\theta), t(\theta))$ is an optimal contract generating equilibrium effort $e(s, b, t)$. Further suppose $t = e - z$ for some $z > 0$. Call this $t_L$. Let $t_H = e + z$. Then we have,

$$C(\gamma(\theta)) = g(\gamma(\theta) + e(\theta)) = g(z) \Delta = g(-z) \Delta = g(\gamma(\theta) + e(\theta) - t_H(\theta)) \Delta \tag{63}$$

The first inequality comes from (IC), the second from the definition of $t_L$, the third by symmetry\textsuperscript{12} of $g$, and the fourth by the definition of $t_H$. Therefore, $t_H$ also satisfies (IC). The same argument works if $z < 0$.

\textsuperscript{11}Michal Matejka provided this proof.

\textsuperscript{12}This is an example of how symmetry eases the calculation of the targets. If the distribution is symmetric, the targets are equally spaced around equilibrium effort. Without symmetry, the targets would not be equidistant from the equilibrium effort, as long as the distribution is still single-peaked. If the distribution was not even single-peaked, there would be multiple targets.
Lemma 3  If \( t(\theta) = \gamma \theta + e(\theta) - z \) implements equilibrium effort for some \( z \), then so does \( t(\theta) = \gamma \theta + e(\theta) + z \).

Proof of Lemma 3:  Suppose \( (s(\theta), b(\theta), t(\theta)) \) is an optimal contract generating equilibrium effort \( e(\theta) \). Further suppose \( t(\theta) = \gamma \theta + e(\theta) - z \) for some \( z > 0 \). Call this \( t_L(\theta) \). Let \( t_H(\theta) = \gamma \theta + e(\theta) + z \). Then we have,

\[
C'(e(\theta)) = g(\gamma \theta + e(\theta) - t_L(\theta)) \Delta = g(z) \Delta = g(-z) \Delta = g(\gamma \theta + e(\theta) - t_H(\theta)) \Delta.
\]  (64)

The first inequality comes from \( (EIC) \), the second from the definition of \( t_L \), the third by symmetry of \( g \), and the fourth by the definition of \( t_H \). Therefore, \( t_H \) also satisfies \( (IC) \). Similarly if \( z < 0 \).

Proof of Proposition 4:

Part One.  We have

\[
e(\theta) = \frac{MB(\theta)}{c} = \frac{b(\theta) g(\gamma \theta + e(\theta) - t(\theta))}{c}.
\]  \( (EIC) \)

By Lemma 3, there will be at least two solutions \( (t_L, t_H) \) to this equation, because the density \( g \) is symmetric around zero. Now \( t_H > t_L \), so \( G(e - t_H) < G(e - t_L) \). The manager receives a larger expected bonus with the low target, so the firm can afford to offer a lower salary \( (s_L < s_R) \) to guarantee participation and leave incentives unchanged:

\[
EU(s_L, b, t_L) = EU(s_R, b, t_H) = \bar{u}
\]  (65)

for two contracts \( (s_L, b, t_L) \) and \( (s_R, b, t_H) \). Thus both contracts satisfy \( (PC) \), and generate the same equilibrium effort. Because the firm is risk neutral, it is indifferent between a low target that provides a high expected bonus to the agent, and a high target which provides a low expected bonus to the agent.

Consider the high target, which satisfies \( g'(\gamma \theta + e(\theta) - t(\theta)) > 0 \) for every \( \theta \). Because \( g \) increases only over its negative domain, this means that

\[
\gamma \theta + e(\theta) < t(\theta) \text{ for each } \theta.
\]  (66)

Since the monotonicity condition \( (M) \) will hold in equilibrium, we know

\[
e'(\theta) = \frac{MB'(\theta)}{c} > 0.
\]  (67)

Therefore, by (66), it must be that \( t(\theta) \) is increasing in \( \theta \), since it exceeds the sum of two increasing functions.
Suppose (IC) holds. Differentiate $EU(\hat{\theta}, \theta)$ with respect to $\hat{\theta}$

$$s'(\hat{\theta}) - b(\hat{\theta})g \left( \gamma \theta + e - t(\hat{\theta}) \right) t'(\hat{\theta}) + b'(\hat{\theta})G \left( \gamma \theta + e - t(\hat{\theta}) \right) = 0.$$  \hspace{1cm} (68)

Now, differentiate (68) again with respect to $\hat{\theta}$ to generate the second order condition:

$$s''(\hat{\theta}) - b(\hat{\theta})g \left( \gamma \theta + e - t(\hat{\theta}) \right) t''(\hat{\theta}) + b(\hat{\theta}) \left( t'(\hat{\theta}) \right)^2 g' \left( \gamma \theta + e - t(\hat{\theta}) \right)
- g \left( \gamma \theta + e - t(\hat{\theta}) \right)t'(\hat{\theta})b'(\hat{\theta}) - b'(\hat{\theta})g \left( \gamma \theta + e - t(\hat{\theta}) \right)t'(\hat{\theta})
+ b''(\hat{\theta})G \left( \gamma \theta + e - t(\hat{\theta}) \right).$$ \hspace{1cm} (SOC)

Differentiate (LIC) with respect to $\theta$:

$$s''(\theta) - b(\theta)g \left( \gamma \theta + e - t(\theta) \right) t''(\theta) - b(\theta)t'(\theta)g' \left( \gamma \theta + e - t(\theta) \right) (\gamma - t'(\theta))
- b'(\theta)g \left( \gamma \theta + e - t(\theta) \right) t'(\theta) + b'(\theta)g \left( \gamma \theta + e - t(\theta) \right) (\gamma - t'(\theta)) + b''(\theta)G \left( \gamma \theta + e - t(\theta) \right).$$

Combining with the second order condition (SOC) at $\hat{\theta} = \theta$,

$$(SOC) - b(\theta)t'(\theta)g' \left( \gamma \theta + e - t(\theta) \right) \gamma + b'(\theta)g \left( \gamma \theta + e - t(\theta) \right) \gamma = 0.$$ \hspace{1cm} (69)

$(SOC) < 0$, so (69) implies

$$b'(\theta)g \left( \gamma \theta + e - t(\theta) \right) > b(\theta)t'(\theta)g' \left( \gamma \theta + e - t(\theta) \right).$$ \hspace{1cm} (70)

Since $t'(\theta) > 0$ and $g'(\gamma \theta + e - t(\theta)) > 0$, we have $b'(\theta) > 0$ for each $\theta$. So the bonus is strictly increasing, as is the target.

**Part Two.** Let $MB(\theta) = b(\theta)g \left( \gamma \theta + e(\theta) - t(\theta) \right)$. By the usual arguments, (IC) in program (14) is equivalent to the following conditions:

$$EU'(\theta) = \gamma MB(\theta),$$ \hspace{1cm} (LIC)

$MB(\theta)$ is increasing, \hspace{1cm} (M)

for almost all $\theta \in [\underline{\theta}, \bar{\theta}]$. Observe (EIC) becomes

$$e(\theta) = \frac{MB(\theta)}{c}.$$ \hspace{1cm} (EIC)

The participation constraint requires $EU(\theta) \geq 0$ for all $\theta$. Because the $MB(\theta)$ function is increasing, the conditions above imply that the participation constraint will bind for the lowest type, so $EU(\underline{\theta}) = 0$. By the fundamental theorem of calculus,

$$EU(\theta) = \int_{\underline{\theta}}^{\theta} EU'(x)dx = \int_{\underline{\theta}}^{\theta} \gamma Z(x)dx.$$ \hspace{1cm} (71)
Integrating by parts gives
\[
\int_\theta^\infty E U(\theta)f(\theta)d\theta = \int_\theta^\infty E U'(\theta)d\theta - \int_\theta^\infty E U'(\theta)F(\theta)d\theta = \int_\theta^\infty E U'(\theta)H(\theta)f(\theta)d\theta ,
\] (72)
where \( H(\theta) \) is the inverse hazard rate, defined as \( H(\theta) = \frac{1-F(\theta)}{f(\theta)} \).

Substituting in the virtual utility
\[
EU(\theta) = s(\theta) + b(\theta)G(\gamma \theta + e - t(\theta)) - C(e) ,
\] (73)
we can rewrite this as
\[
\int_\theta^\infty \left[ s(\theta) + b(\theta)G(\gamma \theta + e - t(\theta)) \right] f(\theta)d\theta = \int_\theta^\infty \left[ C(e) + \gamma MB(\theta)H(\theta) \right] f(\theta)d\theta .
\] (74)

Substituting the above equation and the effort incentive compatibility constraint (EIC) into the objective function of program (14) reduces the firm’s problem to choosing the bonus and target to maximize profits subject to the constraint that \( MB(\theta) \) is increasing in \( \theta \):
\[
\max_{(b(\theta), t(\theta))} \int_\theta^\infty \left[ \gamma \theta + \frac{MB(\theta)}{c} - C(e) - \gamma MB(\theta)H(\theta) \right] f(\theta)d\theta .
\] (75)

I use the standard argument and solve the above maximization without the monotonicity constraint, and verify later that the solution to the relaxed problem satisfies this constraint. Rearranging,
\[
\max_{MB(\theta)} \int_\theta^\infty \left[ \gamma \theta - C(e(\theta)) + \left(1 - \gamma H(\theta)\right) \frac{MB(\theta)}{c} \right] f(\theta)d\theta ,
\] (76)
where \( C(e(\theta)) = (MB(\theta))^2 / 2c \). Maximizing the objective function pointwise yields:
\[
MB(\theta) = 1 - \gamma H(\theta) .
\] (77)

When \( 1 - \gamma H(\theta) > 0 \), the optimal value of the bonus is interior for all \( \theta \in [\theta, \bar{\theta}] \), and given by the following first order condition:
\[
b(\theta) = \frac{1 - \gamma H(\theta)}{g(\gamma \theta + e(\theta) - t(\theta))} .
\] (78)

When \( 1 - \gamma H(\theta) < 0 \), the optimal pay-performance schedule is
\[
MB(\theta) = \begin{cases} 
0, & \text{if } \theta \in [\underline{\theta}, \bar{\theta}] ; \\
1 - \gamma H(\theta), & \text{if } \theta \in [\underline{\theta}^*, \bar{\theta}] ;
\end{cases}
\] (79)
where \( \theta^* \in [\underline{\theta}, \bar{\theta}] \) solves \( 1 - \gamma H(\theta^*) = 0 \). Thus
\[
b^*(\theta) = \max \left\{ 0, \frac{1 - \gamma H(\theta)}{g(\gamma \theta + e(\theta) - t(\theta))} \right\} .
\] (80)

Since \( H(\theta) \) is decreasing, we have that (M) holds. ■


