

# An Efficient Revenue Distribution Method in A Cyber-Enabled Manufacturing System

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## Abstract

Considering a group of suppliers fulfilling the orders in an e-commerce platform, it is desirable for the platform operator to distribute the revenue in a way that the total net utilities of the platform and of each supplier are optimized. Based on the game theory, this paper presents a nonlinear revenue distribution rule to achieve Nash Equilibrium. Simulation studies show that, compared to the proportional and equal distribution rules, the proposed method yields the highest net utility for all. The present work demonstrates an approach to promoting broader participation of manufacturing activities in an internet-based platform enabled by cyber infrastructure.

*Key words:* cyber-enabled manufacturing; revenue distribution; game theory

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## Introduction

Manufacturing is typically a capital and skill intensive industry. With the advances in manufacturing technologies and cyber infrastructure, the barrier-to-entry into manufacturing, however, has been significantly eased. Today, it is possible for a company to establish a manufacturing facility with CNCs or 3D printers, conduct business through internet, and supply parts to customers across the country and around the world. Therefore the development of a pervasive manufacturing environment consists of providers and customers of manufacturing technologies, services, and products can be enabled by an internet-based platform. The platform is related to the concept of cloud-based design and manufacturing [1, 2], and can be perceived as the *Uber, Airbnb* or *Amazon* of manufacturing for time sharing of manufacturing equipment (e.g. 3D printers) and transaction of manufactured goods.

For the success of a pervasive manufacturing environment described above, it is critical that the customers seeking for manufacturing services and the participating manufacturers find it beneficial and profitable to use the cyber-enabled platform. This paper focuses on the interaction between the numerous manufacturers and the platform. One way the platform can encourage manufacturers' participation would be to provide each participant with a suitable portion of the revenue obtained from selling to customers. Specifically, a revenue sharing methodology is presented that illustrates how the platform owner could optimally distribute a portion of its revenue among all manufacturing service suppliers to maximize total net utility achieved by the platform and the manufacturers while at the same time optimizing the individual net utility for each manufacturer.

The literature on revenue-sharing is extensive for traditional supply chains and more recently for supply chains under e-commerce [3-6]. However, there is notable lack of theoretic publications on this topic with explicit consideration to characteristics unique to manufacturing service suppliers, and even less in the context of cyber-enabled supply chains. This paper presents initial results aimed at filling this research gap. The work demonstrates an approach to promoting broader participation of manufacturing activities to achieve a cyber-enabled pervasive manufacturing environment.

## Theory

Assuming convexity, the problem of maximizing the total net utility can be easily solved by convex optimization toolboxes when one has complete information of all the reward and cost functions. In practice, however, the cost functions are private information of suppliers, and a strategic supplier may not reveal its true cost function. Therefore, game theoretical approach is necessary.

Consider a cloud platform that connects a massive number of designers/customers and  $n$  manufactured part suppliers, as shown in Figure 1. The platform assigns the arriving tasks to different suppliers to fully utilize available manufacturing equipment/resources. Let  $\mu_i$  denote the performance of the  $i$ -th supplier, which can be measured as the number of basic parts that can be manufactured in unit time. Let  $\boldsymbol{\mu} := [\mu_i]$  denote the performance vector of all suppliers, and  $\boldsymbol{\mu}_{-i}$  denote the performance vector of all suppliers other than the  $i$ -th supplier.

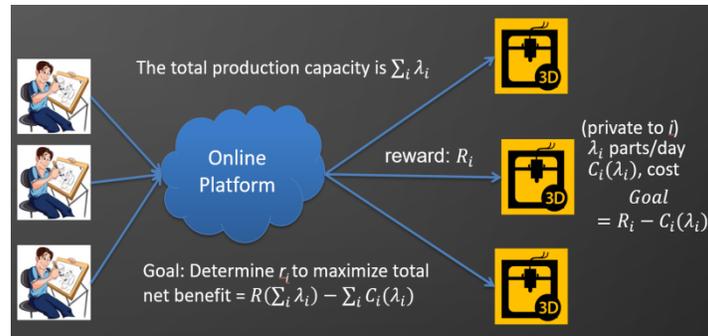


Figure 1. Cyber-enabled manufacturing platform

From the perspective of the cloud platform, the overall system performance is  $\mu_\Sigma := \sum_i \mu_i$ . Suppose such performance generates a revenue of  $R_{\text{tot}}(\mu_\Sigma) := R_0(\mu_\Sigma) + R(\mu_\Sigma)$  to the cloud platform, where  $R_0(\mu_\Sigma)$  is the net profit of the platform itself, and  $R(\mu_\Sigma)$  is the revenue to be distributed to the  $n$  suppliers. For instance, the platform can choose  $R_0(\mu_\Sigma)$  to be a certain percentage of  $R_{\text{tot}}(\mu_\Sigma)$ . It is assumed that the function  $R(\cdot)$  is strictly concave, non-decreasing, satisfies  $R(0) = 0$ , and can be well-fitted by an  $(n - 1)$ -order polynomial  $\hat{R}(\mu_\Sigma)$  via, for example, Chebyshev least squares approximation. Suppose each supplier receives a revenue of  $r_i$

$(\mu_i, \mu_{-i})$  after the distribution. As it is desired to maintain (approximate) budget balance for the revenue distribution,  $\sum_i r_i(\mu_i, \mu_{-i}) \approx R(\mu_\Sigma)$ . The formal definition is as follows:

**Definition 1.** A rule of distributing revenue,  $[r_i(\cdot)]$ , is said to *converge to budget balance in n* if

$$\lim_{n \rightarrow \infty} \sup_{\mu} \left| \sum_{i=1}^n r_i(\mu_i, \mu_{-i}) - R(\mu_\Sigma) \right| = 0 \quad (1)$$

To maintain a certain level of performance  $\mu_i$ , the supplier  $i$  needs to invest on raw materials, energy, device maintenance, and so on, which will incur a cost denoted by  $C_i(\mu_i)$ . The cost functions are assumed to be convex. Now for supplier  $i$ , since the revenue is  $r_i(\mu_i, \mu_{-i})$  and the cost is  $C_i(\mu_i)$ , the *net utility* is therefore  $r_i(\mu_i, \mu_{-i}) - C_i(\mu_i)$ . Each supplier  $i$  aims to maximize its own net utility. As such, the system reaches a Nash Equilibrium if no supplier in the system can improve its own net utility unilaterally.

**Definition 2.** A vector  $\tilde{\mu} := [\mu_i]$  is said to be a Nash Equilibrium if

$$\mu_i = \operatorname{argmax}_{\mu_i} r_i(\mu_i, \mu_{-i}) - C_i(\mu_i), \forall i \quad (2)$$

The platform aims to maximize the total net utility in the system, which can be written as  $\sum_i r_i(\mu_i, \mu_{-i}) - C_i(\mu_i)$ . Due to approximate budget balance, It can be stated that the platform aims to maximize  $R(\mu_\Sigma) - \sum_i C_i(\mu_i)$ . Let  $\mu^* := [\mu_i^*]$  be the vector that maximizes the total net utility, then the platform's problem is to find the rule that distributes revenue,  $[r_i(\cdot)]$ , to induce optimal choices of  $[\mu_i]$ .

**Definition 3.** A rule of distributing revenue,  $[r_i(\cdot)]$ , is said to be *efficient* if  $\mu^*$  is the only Nash Equilibrium.

Before discussing how to find the efficient revenue distribution rule, the necessary conditions for the optimal vector  $\mu^* = [\mu_i^*]$  to be a Nash Equilibrium should be identified. Since by definition, the optimal vector  $\mu^*$  maximizes the total net utility  $R(\mu_\Sigma) - \sum_i C_i(\mu_i)$ , it holds that

$$\frac{\partial}{\partial \mu_i} R(\mu_\Sigma^*) = C_i'(\mu_i^*) \quad (3)$$

On the other hand, if  $\boldsymbol{\mu}^*$  is also the Nash Equilibrium under some revenue distribution rule  $[r_i(\cdot)]$ , then  $\mu_i^*$  maximizes  $r_i(\mu_i, \mu_{-i}^*) - C_i(\mu_i)$ , and thus

$$\frac{\partial}{\partial \mu_i} r_i(\mu_i^*, \mu_{-i}^*) = C_i'(\mu_i^*). \quad (4)$$

Combining the above equations yields

$$\frac{\partial}{\partial \mu_i} [R(\boldsymbol{\mu}_\Sigma^*) - r_i(\mu_i^*, \mu_{-i}^*)] = 0 \quad (5)$$

The above equation suggests that an efficient rule of revenue distribution should ensure that  $R(\boldsymbol{\mu}_\Sigma) - r_i(\mu_i, \mu_{-i})$  is only determined by  $\mu_{-i}$ , and is not influenced by  $\mu_i$ . This implication has indeed been formally stated and proved by Ray & Goldamis [7].

**Proposition 1.**  $[r_i(\cdot)]$  is efficient if and only if there exists functions  $E_i: \mathbb{R}^{n-1} \rightarrow \mathbb{R}$  such that for all  $i$ ,

$$r_i(\mu_i, \mu_{-i}) = R(\boldsymbol{\mu}_\Sigma) - E_i(\mu_{-i}) \quad (6)$$

Recall that  $R(\boldsymbol{\mu}_\Sigma)$  can be fitted by an  $(n-1)$ -order polynomial,  $\hat{R}(\boldsymbol{\mu}_\Sigma) = a_1 \mu_\Sigma + a_2 \mu_\Sigma^2 + \dots + a_{n-1} \mu_\Sigma^{n-1}$ . To better introduce the revenue distribution rule, some helpful terminology is defined:

$$P^j := \left\{ \mathbf{p} = [p_i] \mid p_i \text{ is a nonnegative integer, } \sum_{i=1}^n p_i = j \right\} \quad (7)$$

$$P_i^j := \{ \mathbf{p} \in P^j \mid p_i = 0 \} \quad (8)$$

for  $j = 1, \dots, n-1$  and  $i = 1, \dots, n$ . Next, for  $\mathbf{p} \in P^j$ , let  $G(\mathbf{p})$  be the number of nonzero coordinates of  $\mathbf{p}$ , i.e.  $G(\mathbf{p}) := |\{l \mid p_l \neq 0\}|$ . Note that  $G(\mathbf{p})$  is at most  $j$ , for all  $G(\mathbf{p})$ . Finally, define  $\binom{j}{\mathbf{p}} := \frac{j!}{p_1! \cdots p_n!}$

The designed revenue distribution rule is given as follows. Let

$$\beta_i^j = a_j \sum_{\mathbf{p} \in P_i^j} \frac{n-1}{N-G(\mathbf{p})} \binom{j}{\mathbf{p}} \mu_1^{p_1} \dots \mu_n^{p_n} \quad (9)$$

for  $j = 1, \dots, n-1$ . Let

$$E_i(\mu_{-i}) = \sum_{j=1}^{n-1} \beta_i^j \quad (10)$$

Then

$$r_i(\mu_i, \mu_{-i}) = R(\mu_\Sigma) - E_i(\mu_{-i}) \quad (11)$$

where  $r_i(\mu_i, \mu_{-i})$  is the revenue distributed to each supplier  $i$ .

**Theorem 1.** The rule of revenue distribution  $[r_i(\cdot)]$  as defined above is efficient and converges to budget balance in  $n$ .

Note that if the revenue function  $R(\mu_\Sigma)$  is indeed an  $(n-1)$ -order polynomial, then exact budget balance is achieved. In summary, the amount for each manufacturer participant can be calculated using the following Revenue Distribution Procedure:

Efficient Supplier Revenue Distribution Procedure (**ESRD**):

Step 1. Determine coefficients  $a_1, a_2, \dots, a_{n-1}$  from an  $(n-1)$  polynomial approximation of the given  $R(\mu_\Sigma)$ .

Step 2. Calculate  $\beta_i^j$  and  $E_i(\mu_{-i})$  using expressions (9) and (10), respectively.

Step 3. Determine the revenue distribution for each supplier using expression (11).

## Results and Discussions

Simulation studies were conducted in two example scenarios with high and low performance variations between manufacturing suppliers respectively. Each scenario has  $n = 10$  suppliers. The performance  $\mu_i$  for all suppliers for both scenarios are listed in Tables 1 and 2 respectively. For the high variation scenario, they are generated from the uniform distribution in  $[40, 160]$ . For the low variation scenario, they are generated from the uniform distribution in  $[80, 120]$ .

Both scenarios have a revenue function  $(\mu_\Sigma) = \alpha_0 \sqrt{\mu_\Sigma}$ , where  $\alpha_0 = 1$ . The individual cost functions are  $C_i(\mu_i) = \alpha_i \mu_i^2$ , where the values of  $\alpha_i$ , are also listed in Tables 1 and 2 respectively.

Table 1. Parameters for high variation scenario.

$i$	$\mu_i$	$\alpha_i$
1	64	7.33E-04
2	157	1.23E-04
3	136	1.64E-04
4	137	1.58E-04
5	157	1.23E-04
6	45	1.39E-03
7	81	4.53E-04
8	103	2.68E-04
9	82	4.19E-04
10	58	8.84E-04

Table 2. Parameters for low variation scenario.

$i$	$\mu_i$	$\alpha_i$
1	100	2.78E-04
2	82	4.56E-04
3	115	2.30E-04
4	117	2.26E-04
5	86	3.93E-04
6	95	3.40E-04
7	103	2.65E-04
8	113	2.25E-04
9	85	4.16E-04
10	115	2.15E-04

The efficient revenue distribution rule was compared against two practical baseline rules. One is a *proportional distribution rule (PDR)*, where each supplier  $i$  receives its revenue proportional to its performance, i.e.  $r_i(\mu_i, \mu_{-i}) = \frac{\mu_i}{\mu_\Sigma} R(\mu_\Sigma)$ . The other is a simple *equal distribution rule (EDR)*, where each supplier  $i$  receives an equal share of distributed revenue, i.e.  $r_i(\mu_i, \mu_{-i}) = R(\mu_\Sigma)/n$ .

Figure 2 summarizes the simulation results for revenue, cost, and total net utility at Nash Equilibrium for the high variation scenario under the three proposed revenue distribution rules.

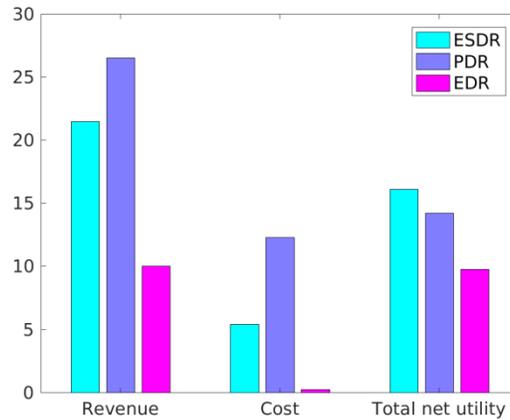


Figure 2. Results of the simulation

As expected, the proposed ESDR is optimal yielding the highest net utility when compared with the PDR and EDR. PDR yields the highest revenue but at the expense of higher costs; this is a proportional revenue distribution rule that the resulting distribution is biased by revenue. On the other hand EDR yields the minimum cost but at the expense of suboptimal revenue. The results for the low performance variation data were similar, and the results are omitted for brevity.

The calculated distributed revenues for each manufacturer for high and low variation scenarios are summarized in Figures 3 and 4. It can be observed that the proposed ESDR adjusts the appropriate revenue for each manufacturer according to their performance. Interestingly, the revenue of each manufacturer depends on both its own performance, as well as the performance of others. As a result, the ESDR promotes the cooperative nature of the cyber-enabled manufacturing environment, since it incentivizes manufacturers to adopt a strategy that is beneficial to the overall environment.

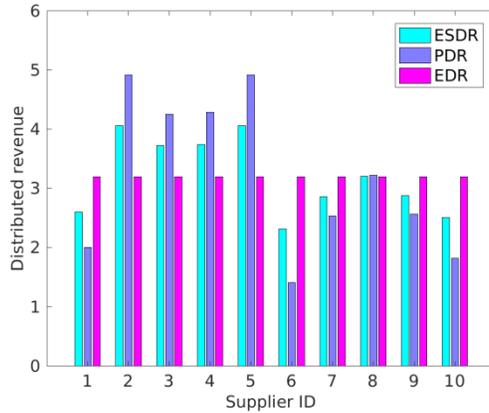


Figure 3. Distributed revenue for high variation scenario.

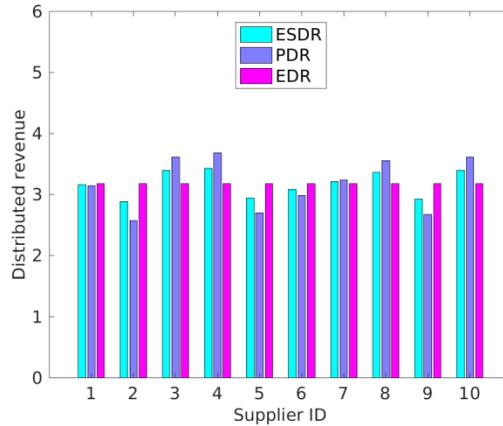


Figure 4. Distributed revenue for low variation scenario.

From the platform operator’s point of view, ESDR can be viewed as a non-profit model since it optimizes the total net utility for the system (platform and manufacturers) and each of the manufacturers. This case may be appropriate if the owner is a regional or government agency, or may occur in early stages of implementation when government or other non-profit organizations are trying to promote the use of the platform and facilitate the development of cyber-enabled manufacturing industry. Also, interesting is for-profit case, in which a private organization owns the platform; it is conjectured that the proposed approach can be modified to consider this case by allowing the platform owner to keep a portion of the revenue.

## Conclusions

With the advances in manufacturing technology and cyber infrastructure, it is conceivable that a cloud platform can be developed to connect customers and suppliers of manufacturing services in a way similar to a ride-share platform connecting riders and drivers. For such a pervasive cyber-manufacturing system to succeed, a massive number of customers and suppliers is required. This paper presents a nonlinear revenue distribution method to promote broad participation of suppliers. Based on the game theory approach, the method optimizes the net utility of the platform and individual suppliers to achieve the Nash equilibrium of the system. The simulation results show that, with the proposed method, the revenue distribution depends not only on individual supplier's performance, but also on the performance of other participants in the system. One future enhancement of the present work is to develop fast approximations that will address scalability issues and allow online implementation with a large number of suppliers.

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## References

- 1 Wu, D., Rosen, D.W., Wang, L., & Schaefer, D. (2015). Cloud-Based Design and Manufacturing: A New Paradigm in Digital Manufacturing and Design Innovation, *Computer-Aided Design*, 59(1): 1-14, doi:10.1016/j.cad.2014.07.006.
- 2 Wu, D., Greer, M.J., Rosen, D.W., & Schaefer, D. (2013). Cloud Manufacturing: Strategic Vision and State-of-the-Art, *Transactions of the SME, Journal of Manufacturing Systems*, 32(4): 564-579. doi:10.1016/j.jmsy.2013.04
- 3 Cachon GP and MA Lariviere, "Supply Chain Coordination with Revenue-Sharing Contracts: Strengths and Limitations," *Management Science* (2005), 51 (1), pp. 30-44.
- 4 Kunter M, "Coordination via cost and revenue sharing in manufacturer-retailer channels," *European Journal of Operational Research*, *European Journal of Operational Research* (2012), 216, 477-486.
- 5 Calvo E and V Martínez-de-Albéniz, "Sourcing strategies and supplier incentives for short-life-cycle goods," *Management Science* (2016), 62 (2), 436 -455.

- 6 Qihui Lu and Nan Liu, "Pricing games of mixed conventional and e-commerce distribution channels," *Computers & Industrial Engineering* (2013), 64 (1), 122-132.
- 7 K. Ray and M. Goldmanis, "Efficient cost allocation," *Management Science*, vol. 58, no. 7, pp. 1341–1356, 2012.