Optimal Performance Targets*

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Abstract

I study a class of contracts in which the manager earns a discrete bonus if his performance clears an explicit threshold. These performance targets provide the firm with an additional instrument to resolve its moral hazard problem with its manager. The performance target easily achieves first best under risk neutrality, setting the target precisely equal to the desired effort that the firm seek to induce. The optimal bonus increases in risk, which may help explain the weak empirical support for the risk-incentives trade-off. If the manager is risk averse, the firm will shade the optimal target below equilibrium effort to provide a form of insurance to the manager, outside of the standard reduction in the bonus. This is consistent with empirical and survey data that suggest that firms set targets to be achievable for their managers.

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1 Introduction

Managerial compensation often takes the form of a bonus and a performance target, in which a manager earns a bonus if his performance clears an explicit threshold.[1] However, the theoretical literature has either examined the class of linear contracts under normally distributed errors and exponential utility (the so-called LEN model), or has articulated optimal nonlinear contracts in full generality that bear little resemblance to contracts used in practice. What is missing is a theoretical exploration of performance targets to provide both positive prediction and normative guidance.

I examine the class of performance target contracts under a variety of settings. To fix ideas, I begin with a risk-neutral agent, and show that performance targets easily achieve first-best, with the target optimally set to efficient effort. This result bears similarity to the efficiency of rank-order tournaments. And for good reason, since a performance target is like a tournament, except that the target is not a strategic choice by a separate agent, but rather an optimal choice of the firm. The target provides an extra contract parameter, so that the firm can keep one of the other compensation parameters (the bonus) fixed. Thus, the target offers the firm an additional instrument to resolve the manager's effort problem, a theme that will permeate the analysis. Contrast this to this linear contract, in which salary and bonus both change when the environment changes.

I then examine risk. The linear model predicts that increases in risk will result in smaller bonuses, as the bonus loads risk onto the manager. However, the empirical evidence on the risk-incentives trade-off has been mixed (Prendergast 2002). Under a performance target, the optimal bonus increases in variation in the manager's performance measure. When output variance increases, this dampens incentives to work, as it is less likely that output from a given unit of effort will clear the target (because of the increased noise in the system). To compensate for this, the firm increases the bonus in order to extract effort out of the manager. That the bonus increases in noise may help explain the mixed empirical tests of the risk incentives trade-off. These tests largely regress pay-performance sensitivity (PPS) on stock return volatility, and measure PPS through changes in total direct compensation. Here, an increase in risk directly increases

[1] Such bonus targets contracts are common in among managers in sales (Oyer 2000, ECS 1995), commercial banking (Simons and Davila 1997), investment management (Perold and Stafford 2012), manufacturing (Narayanan et al 2003), and franchising (Brickley et al 1991). They also appear in executive contracts disclosed in corporate filings, usually as bonuses paid directly in cash or in performance shares that ultimately convert to cash.

the bonus, which will increase direct compensation and therefore PPS, providing a partial explanation for why the empirical tests of the risk incentives trade-off have been mixed.

Next, I solve the model under general risk aversion. While it would be efficient to pay the manager a flat wage in order to provide full insurance, this would ruin incentives to work. I show that the firm will optimally select a target below the second-best equilibrium effort level. Just as the second-best program involves a smaller bonus to reduce the manager's exposure to risk, so does the lower target provide this insurance effect to the manager. Once again, the target serves as a substitute instrument for the bonus, as they alternatively resolve the manager's moral hazard problem with the manager.

There will always be two solutions that induce the same effort, given by a low target and a high target. However, even though both targets implement the same effort, the firm is not indifferent. The low target is easier to achieve, and therefore, the manager is more likely to receive his bonus, so his expected bonus compensation is higher. Because of this, he will accept a smaller salary to participate. Because of risk aversion, the firm can lower the bonus also to match incentives at the high target. As such, the firm prefers the low-target contract because it can induce identical effort at lower cost.

I focus attention on finding the optimal contract within the class of performance target contracts; I do not solve for the optimal contract under all possible contracts to show that performance targets are optimal. First, other papers already seek to examine the question of global optimality, which I cite below. But also, there may be exogenous reasons why firms select target/bonus contracts that are beyond the scope of this analysis (contracting constraints, institutional history, etc). The prevalence of these contracts (see Footnote 1) suggests that these constraints bind. I still find it useful to understand how firms design contracts *within* these exogenous constraints. I write in the spirit of Ross (2004), who urges research to consider properties of contracts that are used in practice, rather than focusing attention exclusively on fully general contracts that are mathematically complex but lack realism. I depart from the LEN model in my focus on targets (not linear), general forms of risk aversion (not only exponential) and general distributions that are symmetric and unimodal (not only normal).

There's a small empirical literature on performance targets and an even smaller theoretical one. Murphy (2001) is an early empirical analysis of performance targets that finds that internally-determined performance standards are more likely to have discontinuous features that lead to income smoothing. Murphy (2001) considers compensation in the form of $s + b(X - \bar{X})$, where X is the manager's performance measure, and $\bar{X}$ is the standard that the manager faces. While this does capture the flavor of a performance that must exceed a standard, it is nonetheless a linear contract in $X - \bar{X}$. Indeed, much of the prior literature assumes targets of this form, and does not model the discontinuous nature of the target explicitly. This paper aims to use the description of performance targets and standards from Murphy (2001), but to model the manager's optimization problem more explicitly.

Murphy (2001) further documents the presence of an "incentive zone" in which the manager's pay is linear within the incentive zone, and flat outside of it. I do not consider the optimal incentive zone, as it is less common outside of top executive contracts, and explored elsewhere. Gutiérrez Arnaiz and Salas-Fumás (2008) show that the incentive zone collapses to a "dichotomous bonus" (the kind I consider here) when the performance horizon collapses, say from an annual basis to a quarterly basis.

Gutiérrez Arnaiz and Salas-Fumás (2008) solve for the optimal contract in a specific setting.[2] They do not answer the more general question of optimal targets under any symmetric distribution. Their contract is curvi-linear in the incentive zone, as it is a function of the likelihood ratio, a standard feature of optimal contracts under risk aversion. This function is convex and then becomes concave after it hits an inflection point, which the authors argue is effectively the performance target. However, because they solve their model in a general continuous framework, they do not have a precise characterization of the optimal bonus and target.

Hemmer (2013) examines a special distribution, the modified Laplace distribution, and finds that the optimal contract takes the form of the incentive zone. Both paper Hemmer (2013) and Gutierrez and Salas-Fumas (2008) examine a general contract space with a restricted distribution. Here, I consider a general distribution but a restricted contract space. Both approaches are similar in that they make assumptions to generate precise results.

Other theoretical work on targets examines stage financing in venture capital (Dahiya and Ray 2012) and performance evaluation over multiple periods (Ray 2007). Indjejikian

[2]They assume a Symmetric Variance Gamma (SVG) distribution, and the agent makes a one shot change to the mean of a stochastic process. Madan and Seneta (1987) and Carr, Geman, Madan, and Vor (2002) show that the SVG process fits data from share prices, but as of yet, there is no evidence that SVG fits data from accounting numbers, on which most performance targets are based.

et al. (2014) and Gerakos and Kovrijnykh (2013) both consider earnings targets, and the latter paper indeed contains a formal model. However, none of these papers solve for the optimal target. There is of course a large literature on the ratchet effect (Weitzman (1980), Indjejikian et al. (2014), Aranda et al. (2014), Arnold and Artz (2015), and Bouwens and Kroos (2011)), which primarily concerns dynamic changes in targets over time. These papers often ask whether the ratchet effect exists at all, and generally takes the first period target as given, rather than solving for it optimally.

There is an emerging empirical literature on targets, primarily drawn from hand-collected survey evidence and field studies (Mahlendorf et al 2014, Matejka and Ray 2015, Merchant and Manzoni 1989, Merchant et al 2015). These papers consider a wide mix of contracts for a broad range of managers (middle managers, division heads, CFOs, etc). While the specific details and settings of these papers vary, they all find that firms set their financial targets to be highly achievable. This is consistent with our theoretical result that the firm will set the target for a risk averse manager below his equilibrium effort, thereby making the target achievable.

The distinguishing contribution of this paper is to model the performance target explicitly. The prior theoretical literature only discusses targets in the abstract, and does not explicitly solve for an optimal target as a choice variable of the firm. For example, in most of these prior models, the firm does not explicitly pick the target, but rather the optimal contract is consistent with a contract that includes (or has features of) a target. Moreover, this analysis solves the joint problem of the target along with the payments to the manager, such as the salary and bonus. This provides insight into how the firm uses different contracting parameters, mainly the salary versus the target versus the bonus. In this sense, I contribute to the complementarity literature in organizational design (Milgrom and Roberts 1992), which seeks to understand whether the different contracting instruments available to the firm are complements or substitutes in a multidimensional optimization.

The paper proceeds as follows. Section 2 considers the base model under risk neutrality and discusses the risk-incentives trade-off. Section 3 introduces managerial risk aversion under general utility functions. Section 4 concludes.

2 The Model

Managerial compensation is largely internally determined, so it is difficult to document in large samples the explicit form of contracts for employees mentioned in the introduction, such as lower level sales managers and financial traders. Consider some managerial contracts that are publicly disclosed: executive pay contracts curated from proxy statements of corporate filings. In 2010, McDonald's set a target for operating income at \$7.24 billion. If the CEO hit this target, his payout was \$2,160,000. This target was discrete in that it offered a fixed cash payment if the performance cleared the target and nothing otherwise. Chevron set a target based on invested capital with no performance shares awarded if ROIC fell below 18%, 8% awarded if it exceeded 18%, 40% awarded if it exceeded 20%, and 80% awarded if it exceeded 22% or higher. Roughly 38% of the CEO's compensation was paid in performance shares, delivered in cash.

These are all examples of contracts that contain some kind of discrete performance target. Sometimes the payout rises linearly with performance, in which case the board interpolates a bonus number for performance in between two discrete targets, creating an incentive zone.[3] Nonetheless, even absent interpolation, executive contracts often contain some kind of discrete target. The Incentive Lab database covers the top 750 firms (measured by market capitalization) over 1998 to 2012. There are 112 CEOs that receive a cash bonus based on some discrete performance target, without interpolation, assessed on absolute or relative performance evaluation. To keep the analysis focused, I will examine only a single performance target with no interpolation. To see that this does not arbitrarily restrict the analysis, observe that in the Incentive Lab data, 69% of CEOs have contracts that lack interpolation.[4] Of course, multiple performance targets would be a straightforward generalization of the theory developed here.

[3]In this case, the company simply takes the weighted average of performance, and the weighted average of the payout that matches the weight on the performance. For example, suppose the contract offers the executive b_1 if performance clears t_1 and b_2 if performance clears t_2. If actual performance is $\lambda t_1 + (1-\lambda)t_2$, then the interpolated payout is $\lambda b_1 + (1-\lambda)b_2$.

[4]This is the proportion of unique CEOs from 1998-2012 that have received a grant containing at least one performance target/bonus without interpolation, based on absolute performance evaluation. The corresponding number for relative performance evaluation is 45%

2.1 Risk Neutral Benchmark

A risk neutral principal (the firm) contracts with a risk neutral and effort averse manager (the agent). The manager exerts unobservable effort $e \geq 0$ at a cost of effort $C(e) = 0.5ce^2$, so C is strictly increasing and convex. The manager's performance measure is given by

$$q = e + \varepsilon, \tag{1}$$

where ε follows a unimodal, continuous, and symmetric[5] density g and distribution G with mean 0 and variance σ^2. Call e^* the first best effort, given as the solution to $C'(e^*) = 1$, so $e^* = 1/c$. Observe that total surplus $e^* - C(e^*) = \frac{1}{2c} > 0$, so it is efficient to hire the manager.

The firm offers the manager a contract (t, s, b), where t is the performance target, s is the salary, and b is the bonus, contingent on performance. The manager earns the bonus if performance exceeds the target:

$$Pay = \begin{cases} s & \text{if } q < t \\ s + b & \text{if } q \geq t. \end{cases} \tag{2}$$

This fits the simplest description of a performance target, where performance must exceed a threshold before the manager earns a payment. There is a discontinuity in the manager's payoff, jumping from s to $s + b$, when performance exceeds the target.

The probability that the manager receives his bonus is

$$P \equiv Prob(q \geq t) = Prob(\varepsilon \geq t - e) = G(e - t), \tag{3}$$

since by symmetry of g, $G(x) = 1 - G(-x)$. Observe that the probability increases in effort and decreases in the target:

$$\frac{\partial P}{\partial e} = g(e - t) = -\frac{\partial P}{\partial t}. \tag{4}$$

[5]The symmetry of the error distribution is not necessary, but does dramatically ease calculation. The most common distributions, such as uniform, normal, and fat-tailed distributions like the Cauchy, are all symmetric. Many distributions satisfy this assumption, such as the normal distribution and any bell-shaped, fat tailed symmetric distribution (like Cauchy). The unimodal condition combined with symmetry implies that the mean of the distribution is its maximal point.

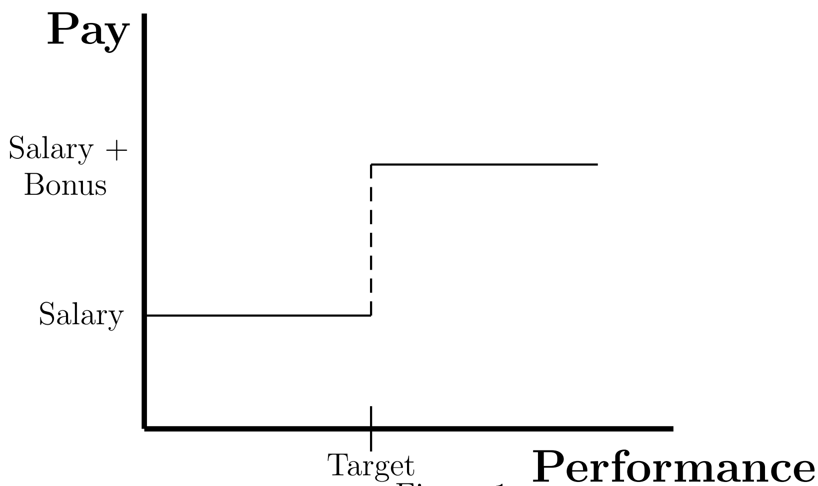

Figure 1

Higher targets directly reduce the manager's probability of achieving his bonus. Target and effort work in exactly opposite directions on this probability. The expected utility of the manager is

$$EU = s + bG(e - t) - C(e). \tag{5}$$

The manager can select effort at cost $C(e)$ to maximize his expected payoff. The solution to this problem generates the incentive constraint for the manager:

$$bg(e - t) = C'(e). \tag{IC}$$

The manager equates the marginal cost of effort to the expected marginal benefit, which is the change in the probability of achieving the bonus, times the unconditional bonus itself. This marginal effect on the change in probability is represented by the term g, and will permeate the analysis. While higher targets unilaterally decrease the probability of clearing the target, the effect on the change in probability is ultimately what matters. Indeed, the firm picks a contract which induces the manager's effort, and the difference between the target and effort will ultimately drive the manager's incentives. As is common, assume the manager faces an outside opportunity $\bar{u}$ in order to induce participation. The manager's expected payoff must exceed this opportunity, and therefore impose the standard participation constraint (PC) that $EU \geq \bar{u}$.

The firm maximizes expected profits, subject to the incentive and participation constraints. The solution to this problem generates the optimal efficient contract.[6] All proofs are in the appendix.

Proposition 1 *The optimal contract that implements first-best effort* e^* *is* (t^*, s^*, b^*) *where*

$$t^* = e^*, \tag{6}$$

$$b^* = \frac{1}{g(0)}, \tag{7}$$

$$s^* = \bar{u} + C(e^*) - \frac{1}{2g(0)}. \tag{8}$$

The proposition solves for the optimal contract, which in this case is efficient. This should come as no surprise, as the manager is risk neutral, and there is no conflict of interest between the firm and the manager. Compare this to the usual linear contract that makes the agent the full residual claimant on firm output, where the firm extracts rents from the manager through a (possibly) negative salary.[7] Here, the efficient contract is nothing like the linear "sell the firm" contract. The firm will set the target to the efficient effort level, and then select the salary and bonus to solve the participation and incentive constraints, respectively. Proposition 1 formally proves that the firm will select the target equal to efficient effort under risk neutrality. This conforms to the common intuition that the target should equal the effort that the principal seeks to induce, which in this case is first-best effort.[8]

The performance target offers a discrete jump in payoff if performance clears the target. Consider this a "prize" of b, the difference in payoff from clearing the target versus not. In equilibrium, $t^* = e^*$, so (IC) in equilibrium becomes

$$b^* = \frac{1}{g(0)}. \tag{9}$$

[6] Oyer (2000) shows that target/bonus contracts are globally optimal over a wide class of contracts for risk-neutral agents, so the restriction to performance target contracts is without loss of generality under risk-neutrality.

[7] The linear contract under risk-neutrality is $b^* = 1$ and $s^* = \bar{u} - \frac{1}{2c}$.

[8] Gutiérrez Arnaiz and Salas-Fumás (2008) offer a numerical example in which the performance standard is set equal to the mode of the performance distribution. But without formally solving for the optimal contract performance target, it is impossible to say for sure whether the target lies above or below equilibrium effort.

The relationship between target and effort is non-trivial, since a shift in the target t will immediately shift equilibrium effort $e(s, b, t)$. However, the proof of Proposition 1 shows that because the manager's participation constraint will bind, firm profits equal total surplus, and therefore the firm can afford to achieve efficiency. Given that the firm seeks to implement e^*, the optimal target will pin down equilibrium efficient effort exactly. This occurs precisely when the returns to managerial effort are highest, the point when a marginal increase in effort leads to the greatest change in probability. This is exactly when the density g hits its maximum at $t^* - e^* = 0$.[9] Moreover, the optimal salary compensates the manager for his outside opportunity and cost of effort, but then deducts half of his bonus from his salary upfront. Indeed, this is necessary in order to provide the manager with strong effort incentives. Comparative statics on the proposition immediately generate the following corollaries. First consider the effect from the changes in the outside options.

Corollary 1 *The optimal bonus is unchanged in the manager's outside options* $\left(\frac{\partial b^*}{\partial \bar{u}} = 0\right)$, *while the optimal salary increases in the manager's outside options* $\left(\frac{\partial s^*}{\partial \bar{u}} > 0\right)$.

The participation constraint ensures that the manager meets his outside options. As with the linear model, increasing outside opportunities forces the firm to increase the salary in order to retain the manager, but make no change in bonus. Now consider changes in the cost of effort, which tracks the quality or productivity of the manager.

Corollary 2 *As the manager's cost of effort increases, the optimal target decreases, the optimal salary decreases, and the optimal bonus is unchanged.*

Because the optimal target is $t^* = e^* = \frac{1}{c}$, it is immediate that the firm will decrease the target as the manager's cost of effort rises. In fact, this is the only comparative static in which the target changes. As effort becomes costly, it is efficient for the manager to work less, as that saves on manager disutility, and therefore social welfare. The corollary shows that even though the firm decreases the target, which decreases effort, it will simultaneously decrease salary. This is a countervailing effect to the fall in the target. Indeed, the target is a powerful instrument and has a direct effect on effort. The salary counteracts this effect somewhat, even though effort will still fall in equilibrium.

[9] The unimodal condition and symmetry imply that the mean of the distribution is its maximal point, which is why the firm will set the difference between target and effort to be equal to this mean of 0.

Compare to the same results from the linear model. In both the linear model and here, the optimal bonus does not change with the manager's cost of effort, but rather the salary changes. However, under risk neutrality, the optimal salary decreases in the cost of effort in the linear model. Recall that the linear contract is a "sell the firm" contract, making the agent the full residual claimant on firm output. As such, the agent captures total surplus, and the firm extracts rents from the manager through the salary. This is why the efficient salary in the linear contract is decreasing in total surplus. As the cost of effort rises, it becomes more costly to employ the manager, and therefore total surplus shrinks. The manager's salary will also fall. Ultimately, the chief reason that the directional effect of c on s varies between the models has to do with the presence of the target. The performance target decreases with c, and this is what allows the optimal salary to decrease. The net effect is that both the salary and target change in the performance target contract, whereas only the salary changes in the linear contract.

There is a close theoretical analogy between rank-order tournaments (Lazear and Rosen 1981) and performance targets. Both rely on a relative comparison of output in order to secure an external prize. In performance targets, that comparison is against an exogenous standard set by the firm, whereas in tournaments that comparison is made against the output of another strategic player in the game. In both models, an increase in risk dampens incentives to provide effort, and both models can implement first best under risk neutrality. Here, the bonus reduces to $g(0)^{-1}$, which in the case of a normal distribution is simply $\sigma\sqrt{2\pi}$, so the bonus increases in risk unambiguously. This feature of how the bonus reacts to a change of risk is quite general, as I show next.

2.2 The Risk-Incentives Tradeoff

There's hardly a more celebrated result in agency theory than the risk-incentives tradeoff. The standard LEN model of linear contracts, exponential utility, and normal errors deviates from efficiency because of the risk premium that the firm must pay the manager to bear risk, through mean-variance preferences that include a disutility for risk. This workhorse model of contract theory, nicely summarized in Prendergast (1999), posits that as risk (measured through the variance of the error distribution) increases, optimal incentives should decrease, since the optimal bonus from that model is $(1 + rc\sigma^2)^{-1}$. Because of this, the firm reduces the optimal bonus away from that which would guarantee efficiency.

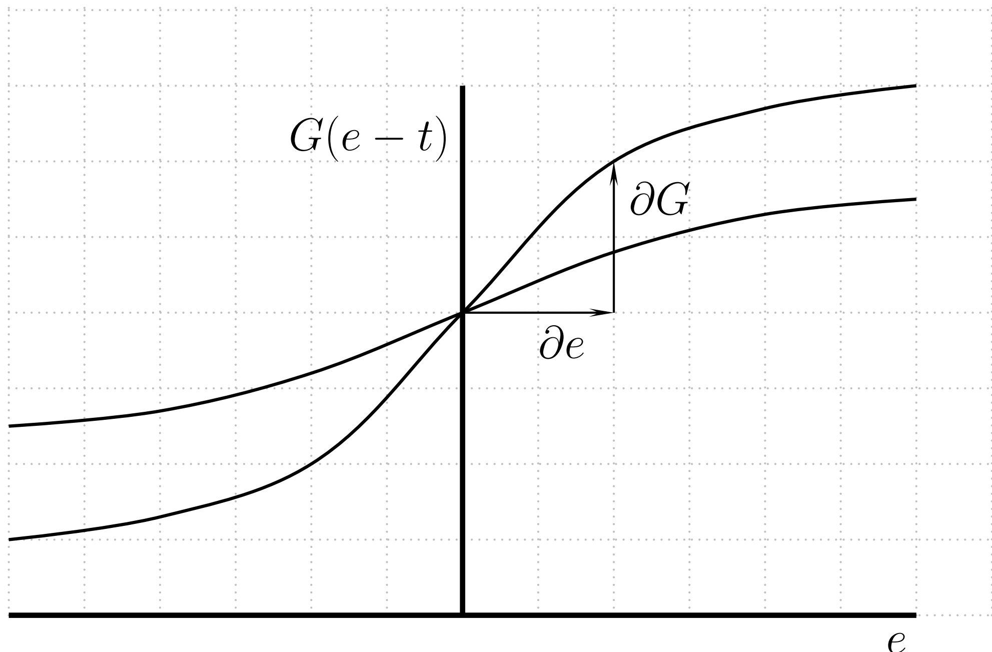

Figure 2: An increase in risk. The steeper CDF second order stochastically dominates the flatter CDF.

The existing literature on the risk incentives trade-off has been mixed. Some papers find a positive relationship (Core and Guay 1999, Oyer and Shaefer 2001, Core and Guay 2002, Nam et al. 2003, and Coles et al. 2006), some find a negative relationship (Lambert and Larcker 1987, Aggarwal and Samwick 1999, and Jin 2002), and some find no relationship at all (Garen 1994, Yermack 1995, Bushman et al. 1996, Ittner et al 1997, and Conyon and Murphy 1999). Most of these papers measure risk as volatility of stock returns and measure incentives as pay-performance sensitivity, measured as changes in direct compensation for a given change in performance. The existing literature has not made a conclusive statement on whether incentives optimally increase or decrease with risk. This calls into question whether the theory is even valid, if it holds under such special circumstances. Indeed, a raft of papers have offered conditions under which the trade-off reverses, giving a positive relationship between risk and incentives (e.g. Dutta 2008 and Prendergast 2002).

Here, the bonus is a reward to the manager for clearing the target, and as the incentive constraint shows, it will equilibrate the marginal cost of effort against the change in probability of clearing the target $g(e-t)$, times the unconditional "prize" of b. Unlike the linear model, there is no disutility for risk that holds over the entire domain of the manager's utility function. Rather, only incentives at the target matter. The assumption of risk neutrality here is to focus on a competing effect, namely the effect of noise on the probability of clearing the target.[10] This effect will still permeate a model of risk aversion, though it may be muted because of the need for insurance. Of course, linear contracts allow no positive relationship for risk and incentives under any conditions.

Proposition 2 *As a variation in the manager's performance measure increases, the optimal bonus increases.*

Said differently, as risk (σ) increases, this dampens the agent's incentives to produce effort. To compensate for this reduction in incentives, the firm must increase the size of the prize, for the same logic as occurs in tournaments.[11] Thus the optimal bonus exactly balances the increase in variance. This fits exactly Proposition 2b of Gutiérrez Arnaiz and Salas-Fumás (2008), who find the same unambiguous result that the bonus size increases in volatility. Even though Gutiérrez Arnaiz and Salas-Fumás (2008) use a more specific model (SVG process), they also find the same reversal of the risk-incentives trade-off.

The term $g(0)^{-1}$ is a proxy for the variance: As the variance on output rises, the tails of the density g will increase and its maximal point $g(0)$ will sink. Recall that G represents the probability of clearing the target, and g is the change in this probability. So, under a higher variance, a marginal change in effort will lead to a smaller change in probability. It is the excess noise that forces the manager to reduce effort. Figure 2 shows two distribution functions, one that is second order stochastic dominant over the other. Remember that a marginal increase in effort changes the probability of achieving the target, and so it is the change in probability (the slope of the distribution function) that matters. In the low variance case, the slope of the distribution is steeper around the mean of 0, so a marginal increase in effort leads to a higher probability of hitting

[10]Prendergast (2002) also assumes risk neutral agents in order to avoid the standard trade-off. It is possible that the trade-off may emerge under risk aversion, though the model does not permit closed forms solutions of this. Instead, numerical simulations later in the paper show that the risk incentives trade-off vanishes under risk aversion.

[11]A colorful analogy in tournaments is the play of tennis (Lazear and Rosen 1981). Under normal conditions both parties exert effort. But imagine if the game occurs during a hurricane that vastly increases the variance on the error term. The ball can go anywhere, and this effectively dampens effort.

the target than under a high variance distribution. Proposition 2 proves this rigorously under two distributions.

Once again, compare this to the linear model in which the optimal bonus is one, and therefore does not change with respect to noise or risk. This may seem like a puzzle, since the incentive contract for a risk neutral manager is fixed under the linear model, but decreases in the performance target model. Yet, inspection of the incentive constraint resolves this difference. Observe that from (IC), the bonus times the change in probability is fixed at one. This reinforces the intuition that the performance target contract includes an extra instrument, namely the target on top of the salary and bonus. Taken together, the expected change in probability times the bonus is the same as the bonus in the linear model. This should not be a surprise, since the expected compensation of a risk neutral agent should not vary with risk, which is true in both models.

There is a natural question of whether the bonus in this model can compare to the bonus in the linear model. Recall that the bonus in the linear model is the slope of the contract, and therefore directly maps into a conceptual definition of pay for performance. Because of the discrete nature of the target, there is no natural analogue to this slope, which is the marginal change in pay for a marginal unit of effort. Nonetheless, the matter is largely immaterial, because empirical estimates of PPS will almost always increase in the discrete bonus of this model.

For example, Aggarwal and Samwick (1999), Jin (2002), and Guay (1999) seek to estimate the risk-incentives trade-off by regressing pay for performance sensitivity (PPS) on risk, usually measured through the volatility of stock returns. Jin (2002) defines PPS as changes in total direct compensation, as well as changes in the re-evaluation of stock and stock options. An increase in the bonus of Figure 1 will increase total direct compensation, and therefore will have an upward effect on the empirical measure of PPS. This will confound the risk-incentives trade-off.

In general, Proposition 2 documents an effect that runs counter to the standard insurance argument. This argument from the linear model claims that the firm will reduce the bonus to provide insurance to a risk-averse manager after an increase in risk. Proposition 2, on the other hand, shows that this increase in risk dampens effort, which the firm will counteract with a higher bonus. Of course, Proposition 2 operated under risk neutrality to isolate the effect, but the effect will still exist under risk aversion (examined in the next section). As such, the true relation between risk and incentives will balance these two effects for a risk averse manager. This fits the claims in Hemmer

(2013) that the relationship between risk and PPS in optimal contracts is not monotone, as most theory currently predicts.

3 Risk Aversion

Now consider that the manager is risk averse and has a utility function u that is strictly increasing and concave. The firm still writes a contract (s, t, b) as before, with a similar bonus and target structure:

$$Pay = \begin{cases} u(s) & \text{if } q < t \\ u(s+b) & \text{if } q \geq t. \end{cases} \tag{10}$$

The discontinuity in the manager's payoff now jumps from $u(s)$ to $u(s+b)$ when output exceeds the target. The expected utility of the manager is

$$EU = \int_{-\infty}^{t-e} u(s)g(\varepsilon)d\varepsilon + \int_{t-e}^{\infty} u(s+b)g(\varepsilon)d\varepsilon - C(e). \tag{11}$$

The integral splits at $t - e$ because that is exactly the point for ε such that the manager earns the bonus ($q \geq t$, or $\varepsilon \geq t - e$). Impose the standard participation constraint (PC) that $EU \geq \bar{u}$. Observe that because the payoff to the manager takes only two discrete values $u(s)$ and $u(s+b)$, the firm can completely control the manager's behavior through the choice of these two payoff levels. As such, the compensation terms pass out of the integral and we can re-write EU as:

$$EU = u(s)G(t-e) + u(s+b)G(e-t) - C(e). \tag{12}$$

Thus, the expectation makes the discrete payoff structure continuous, and so the manager can select effort to maximize his expected payoff. The solution to this problem, given by the first order condition, generates the incentive constraint for the manager:

$$g(t-e)\Big(u(s+b) - u(s)\Big) = C'(e). \tag{IC}$$

As before, the benefit of effort includes its effect on changing the probability of clearing the target, expressed in the term $g(e-t)$. Observe that under risk neutrality, the utility spread collapses to the bonus as a special case. The incentive constraint now contains the term $u(s+b) - u(s)$, which I call the <u>utility spread</u>. This is the gain in

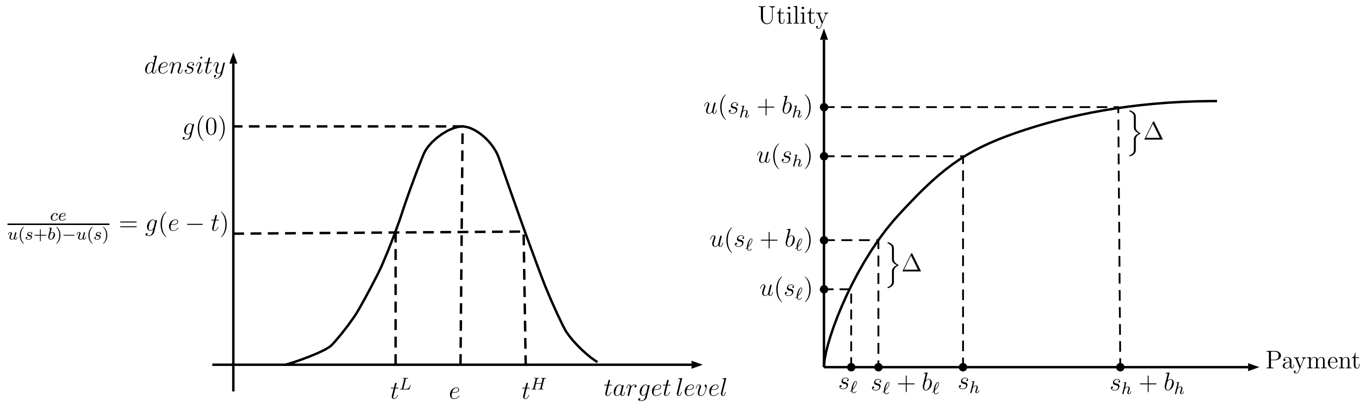

Figure 3: (a): Dual performance targets. (b): Optimality of lower target.

utility from achieving the bonus. Since utility is increasing, the spread rises in the bonus ($u'(s+b) > 0$). Immediate comparative statics on the incentive constraint generate the following results.

Corollary 3 *Equilibrium effort rises in the bonus, falls in the salary, and rises in the target if effort exceeds the target ($\hat{e} > \hat{t}$).*

The first effect from bonus is the same as the canonical model: higher pay-for-performance sensitivity (PPS) induces the manager to work more. The third result is more surprising. In the LEN model, salary has no effect on effort incentives. Indeed, that is largely why the firm can hold the manager to his participation constraint, since it can lower salary without affecting (IC). But now, salary affects the utility spread and therefore incentives. Under a general form of risk aversion, risk preferences at any point depend on wealth at that point. Specifically, because the manager has concave utility and therefore diminishing marginal utility, for a fixed bonus, a higher salary will cause the utility spread to shrink (since $u'(s+b) < u'(s)$). This will decrease effort incentives.

The first best solution maximizes total surplus, the profits of the firm plus the utility of the manager. Because the manager is risk averse, the compensation terms do not fall out of the surplus function, as they did in the risk neutrality case. Now, the social planner maximize surplus subject to the participation constraint. In the bonus and target setting, we arrive at the usual benchmark: if the firm can contract on effort, it will provide full insurance to the agent:

Corollary 4 *Under risk aversion, the first best gives full insurance of the manager, with a flat salary given by $u'(s^*) = \frac{1}{\lambda}$, where λ is the multiplier on the participation constraint.*

The flat salary results from optimal risk-sharing, since the firm is risk neutral and the manager is risk averse. This is the Borch condition that makes equal the marginal utility of the principal and agent. Since the principal is risk neutral, his marginal utility is 1. The term λ in the solution is the multiplier on (PC), and $\lambda > 0$ since (PC) binds in equilibrium.

The concavity of the utility function prevents first best and will lead to an effort distortion. To see this, imagine that the firm could implement first best effort. Plugging this into the incentive constraint generates $g(e-t)\Delta = 1$, where Δ is the utility spread. Rearranging terms gives

$$g(e-t) = \frac{1}{\Delta} > \frac{1}{b^*} = g(0) \tag{13}$$

where the bonus on the right hand side is set at the first best level, and $\Delta < b^*$ by risk aversion. But of course, this is impossible since the distribution peaks at zero. So, in fact, (IC) will hold at an effort level distorted away from first best. This occurs precisely when the utility spread is smaller than the optimal bonus, which must occur since the manager is risk averse and the optimal target lies away from the efficient effort $(t < e^*)$. Indeed, both the utility spread falls short of the bonus, and the change in probability lies beneath its maximal point. And thus the marginal benefit is less than the first best marginal cost of one, yielding the effort distortion.

The firm maximizes expected profits, subject to the incentive and participation constraints. The full program involves expected profits less a multiplier for both constraints:

$$\max_{(s,b,t)} e - (s + bG(e-t)) \tag{14}$$

subject to

$$u(s) + \Big(u(s+b) - u(s)\Big)G(e-t) - C(e) \geq \bar{u}, \tag{PC}$$

$$g(e-t)\Big(u(s+b) - u(s)\Big) = C'(e). \tag{IC}$$

Proposition 3 solves for this program and we discover that the target plays an important role in balancing the risk and incentives problem:

Proposition 3 *Under risk aversion, the incentive and participation constraints both bind. The optimal target is achievable* $(\hat{t} < \hat{e})$*.*

The primary result is that the firm will shade the target downward to handle the manager's risk aversion. Proposition 3 shows that the target, in addition to the bonus, also offers insurance. This removes the insurance burden from the bonus and onto the target, as often occurs when the firm has multiple instruments to design optimal compensation. Recall under the benchmark model that the incentive constraint equalizes the marginal cost of effort against its marginal return. In the structure of this model, that is equivalent to a horizontal line passing through the distribution of effort, as shown in Figure 3.

This occurs because of the symmetry of the error distribution. From (IC), the marginal cost of effort must equal the marginal return, which is the marginal change in the probability of clearing the target times the size of the prize, the utility spread. Because g is symmetric, there will always be two targets symmetrically distributed around equilibrium effort that solve (IC). To see this visually, imagine a horizontal line passing through the density g. The coordinates of the x-axes of the intersection points are the optimum targets that satisfy (IC). There will always be two solutions to this problem, as Figure 3 illustrates.

The low and high targets will equivalently induce the same equilibrium effort. Recall that the probability of clearing the target, P, decreases in the level of the target; as such, the manager has a lower chance of receiving the bonus with high targets. Therefore, the manager receives a higher expected bonus from a low target rather than a high target, so he requires less salary in order to participate. Said differently, the principal must pay a premium to the manager in order to induce participation under a high target. Since both targets generate the same equilibrium effect, the high target has no benefit for output, only a higher cost to induce participation.

This result follows fundamentally from risk aversion. Recall that the utility spread is the difference in utility from receiving the bonus versus just receiving the salary alone. For any fixed bonus, this spread falls in the salary level because of diminishing marginal utility (driven by the risk aversion, illustrated in Figure 3(b)). Therefore, when the firm offers a high target with a low probability of payout, it must offer a corresponding high salary to guarantee participation. That high salary, call it s_H, paired with a given bonus, call it b_H, determines the utility spread and therefore effort incentives. A low target raises the probability of payout, and the firm can afford to pay a lower salary to guarantee participation. Because of diminishing marginal utility for a fixed bonus b_H, the utility spread at the low salary will exceed the utility spread at the high salary, since the utility curve is steeper at the lower salary level. To keep incentives unchanged, the firm can therefore lower the bonus to some $b_L < b_H$, which will match exactly the utility spread and therefore the incentives at the prior contract. To see this visually, observe in Figure 3(b) that the diminishing marginal utility (risk aversion) forces $b_L < b_H$ in order for incentives (Δ) to be identical at both contracts. Thus, the low target pairs with a low salary and low bonus, and offers the same incentives as the high target with a high salary and high bonus, inducing identical effort at lower cost.

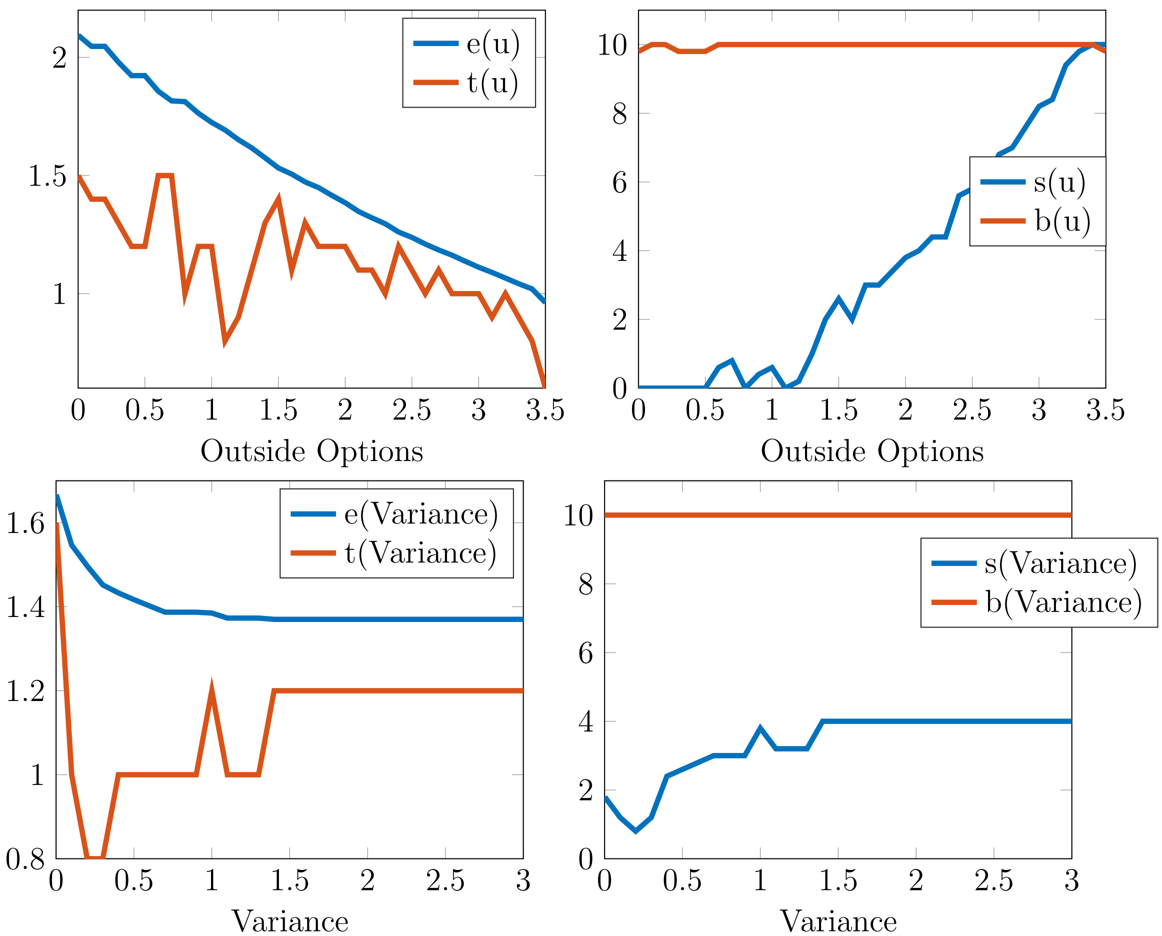

Figure 4: Numerical simulations under quadratic utility and mean-zero normal errors. The top graphs show the optimal contract as a function of the outside option, when variance is fixed at 1. The bottom graphs show the optimal contract as a function of variance, when the outside option is fixed at zero.

As such, in every equilibrium we always have $\hat{t} < \hat{e}$. The full solution in the proof of

Proposition 3 gives the optimal contract as a function of the constraints of the problem, namely, the two Lagrangian constraints on the incentive and participation constraints. The program is inherently complex as salary, bonus and target jointly and simultaneously determine effort. It is impossible to change one variable alone without changing others as well. Thus, the LEN logic, where a change in salary will not affect the incentives, no longer holds. Still, even with this complexity, I show that the participation constraint will always bind. The firm will always select the target such that it extracts the full rent out of the manager.

To see why, observe that the firm has at its disposal a target that can always serve as an extra instrument to modulate the manager's expected utility. If the target is strictly below equilibrium effort, so $\hat{t} < \hat{e}$, then a slack participation constraint leaves rents for the manager. But the firm can always simultaneously lower the salary and raise the target, keeping equilibrium effort constant. Lowering the salary will tighten the participation constraint, since the manager's expected payoff will fall. Raising the target will further tighten the participation constraint, as the manager certainly prefers low targets to high targets. But both these actions will raise effort and, therefore, profits for the firm, and thus allow the firm to implement the same effort at a lower cost (a lower salary and lower expected compensation). The firm will do this until the participation constraint binds.

Figure 4 shows numerical simulations from an example with quadratic utility and normal errors. In the top graphs, I show the optimal contract as a function of the outside options, holding the variance to 1. In the bottom graph, I show the optimal contract as a function of the variance, setting the outside option to zero. Notice that in both graphs, the optimal target lies below equilibrium effort. The optimal bonus is $b = 10$, suggesting that the insurance effect that dampens the optimal bonus exactly balances the effect from Proposition 2. Thus, in this case with quadratic utility, there is indeed no risk incentives trade-off, as the optimal bonus is constant as a function of risk.

4 Conclusion

Until recently, theory has largely ignored what managerial contracts actually look like. In the face of such lack of knowledge about these specific contracts, linear models are good first approximations, given their simplicity and robustness. This has generated the large LEN literature in accounting and finance (on the theory side) coupled with linear tests of the risk incentives trade-off (on the empirical side). Yet, the empirical tests of the risk incentives trade-off remain weak, and a comprehensive test that combines models of actual contracts, with a precise fit to empirical data remains elusive.

A common feature of managerial contracts is the reliance on a performance target of some kind, which involves an explicit payout according to a pre-specified target. This paper models such contracts explicitly and generates a host of new intuitions and insights that can be tested against executive pay data: (1) The optimal bonus increases in risk, (2) the target provides insurance (in addition to the bonus) to help resolve the managers moral hazard problem, and (3) the optimal bonus and target increase in the ability of the manager.

The field is ripe for further explanation and exploration of contracts that firms actually use.[12] The trend towards more disclosure makes these contracts available to the analyst, who can then tailor the theory and generate more precise empirical predictions than were possible before. This new research agenda mixes theory with empirics at a more intimate level, since the contract itself emerges from practice. I remain optimistic for how future work can explore dynamic effects, earnings management, the informativeness principle, team incentives, and many of other theoretical questions of contracts used in practice, of which performance targets are just one example.

5 Appendix

Proof of Proposition 1: Consider a contract (t, s, b). Maximizing expected payoff to the manager with respect to effort gives the incentive constraint (IC). Assume the second order condition holds:

$$-g'(t-e^*)b - C''(e^*) < 0. \tag{SOC}$$

Because both parties are risk neutral and the firm is the contract designer, the firm's expected profit will equal expected total surplus. Therefore, it is possible to implement first best. To do so, first note that $C'(e^*) = 1$. We seek to implement first best effort

[12] An open question is the firms desire to discourage earnings management. A full exploration of this phenomenon is outside the scope of this paper but worthy of future research. Bizjak et al. (2014) finds empirically that performance targets induce real earnings management rather than accruals management, but to date there is no theoretical investigation of this question.

e^*, which satisfies $C'(e^*) = 1$. Plug in $t^* = e^*$ and $e = e^*$ into (IC), and call this the equilibrium incentive constraint:

$$b^* = \frac{1}{g(0)}. \tag{15}$$

The participation constraint must hold:

$$EU = s + bG(e - t) - C(e) \geq \bar{u}. \tag{16}$$

Now $G(e^* - t^*) = G(0) = \frac{1}{2}$, so this becomes

$$\frac{1}{2}(2s + b) \geq \bar{u} + C(e^*). \tag{17}$$

Lowering the salary by a small amount will maintain the participation constraint but not affect the incentive constraint, and therefore increase firm profits. So, (PC) binds. Plugging in (15) above into (PC) gives

$$s^* = \bar{u} + C(e^*) - \frac{1}{2g(0)}. \tag{18}$$

Therefore the contract (t^*, s^*, b^*) implements e^* and satisfies (SOC), (IC) and (PC).
■

Proof of Proposition 2: I use second order stochastic dominance to measure an increase in the dispersion of the distribution. Assume g_i for $i = 1, 2$ are two probability densities over the real line with mean 0 and finite variance that both satisfy the single-peaked condition. Suppose G_1 is second order stochastic dominant over G_2. By definition, for all $w \in (-\infty, \infty)$, we have $S_1(w) < S_2(w)$ and $S_1(\infty) = S_2(\infty)$ where

$$S_i(w) = \int_{-\infty}^{w} G_i(w)\mathrm{d}w. \tag{19}$$

Suppose $g_2(0) > g_1(0)$. By the single-peaked condition, g is increasing over its negative domain, so $g_i(0) > g_i(x)$ for each $x < 0$. Now $g_2(0) > g_1(0)$, both densities are strictly increasing and they both integrate to the same value, $G_1(0) = G_2(0) = \frac{1}{2}$, at the end point of the interval $(-\infty, 0)$. Then there exists a $z \in (-\infty, \infty)$ such that

$$g_2(x) < g_1(x), \ \forall x < z. \tag{20}$$

Integrate both sides of this inequality over $(-\infty, x)$ for each $x < z$ to generate

$$G_2(x) < G_1(x), \ \forall x < z. \tag{21}$$

Integrate over $(-\infty, z)$ to arrive at

$$S_2(z) < S_1(z). \tag{22}$$

This contradicts the definition of SOSD. Therefore $g_2(0) < g_1(0)$, and so the optimal bonus from Proposition 1 is

$$b_1^* = \frac{1}{g_1(0)} < \frac{1}{g_2(0)} = b_2^*. \tag{23}$$

■

Proof of Corollary 3: For shorthand, denote

$$\Delta := u(s+b) - u(s) > 0 \text{ and } \Delta' := u'(s+b) - u'(s) < 0 \tag{24}$$

because utility is increasing concave. Recall that the incentive constraint is

$$g(e-t)[u(s+b) - u(s)] = C'(e) = ce. \tag{IC}$$

Write the second order sufficient condition from the manager's effort problem as

$$g'(e-t)\Delta - c < 0. \tag{SOC}$$

Differentiate (IC) with respect to the bonus:

$$\frac{\partial e}{\partial b} = \frac{g(e-t)u'(s+b)}{c - \Delta g'(e-t)} > 0. \tag{25}$$

This occurs because u is increasing and from (SOC). Now, differentiate (IC) with respect to salary:

$$\frac{\partial e}{\partial s} = \frac{g(e-t)\Delta'}{c - \Delta g'(e-t)} < 0, \tag{26}$$

from (SOC). Finally, differentiate (IC) with respect to the target:

$$\frac{\partial e}{\partial t} = \frac{-g'(e-t)}{c} \geq 0 \text{ iff } t \leq e, \tag{27}$$

because g is increasing over its positive domain and decreasing over its negative domain. From the proof of Proposition 3, we know $\hat{t} < \hat{e}$ in equilibrium and therefore the derivative above is always positive in equilibrium.

■

Proof of Corollary 4: Suppose the firm can contract directly on effort. Then there is no incentive constraint but only a participation constraint. The firm maximizes profit subject to participation, or solves:

$$\max_{(s,b,t)} e - (s + Gb) + \lambda(u(s) + \Delta G). \tag{28}$$

Differentiating with respect to salary gives

$$\lambda(u'(s) + \Delta' G) = 1. \tag{29}$$

Differentiating with respect to the bonus gives

$$u'(s+b) = \frac{1}{\lambda}. \tag{30}$$

Combining these two equations shows

$$u'(s) = u'(s+b) = \frac{1}{\lambda}. \tag{31}$$

Because the utility function is strictly monotonic, this means $b = 0$. Total surplus of the firm is

$$e - (s + Gb) + u(s) + \Delta G - C(e). \tag{32}$$

Maximizing with respect to effort gives the first best level $e^* = \frac{1}{c}$. Therefore we can implement first best with the following contract: $t^* = e^* = \frac{1}{c}$, $b^* = 0$, $u'(s^*) = \frac{1}{\lambda}$.

■

Lemma 1 *If $t \leq e(\hat{s}, \hat{b}, \hat{t})$ for an optimal contract $(\hat{s}, \hat{b}, \hat{t})$, then (PC) binds.*

Proof of Lemma 1: Suppose (PC) is slack. Consider $\hat{t} < e(\hat{s}, \hat{b}, \hat{t})$. Then the firm can raise the target by a small amount, reducing expected utility and maintaining (PC) because it is slack. This raises effort because effort exceeds the target and g is increasing over its negative domain $\left(\frac{\partial e}{\partial t} > 0 \text{ by (27) since } t < e(s, b, t)\right)$. So effort rises, as does expected output and the firm's expected profit. This contradicts that $(\hat{s}, \hat{b}, \hat{t})$ was optimal.

Suppose $\hat{t} = e(\hat{s}, \hat{b}, \hat{t})$. Consider some salary $s' < \hat{s}$. This tightens (PC) by (52) and raises effort by (26) so $e(s', \hat{b}, \hat{t} > e(\hat{s}, \hat{b}, \hat{t}) = \hat{t}$. Raising the target will raise effort by (27), so increase $\hat{t}$ to t' such that $e(s', \hat{b}, t') = t'$.

This is possible since $\frac{\partial e}{\partial t} < 1$ by (48). This further tightens (PC) by (48) and raises profits since output is higher (from effort), salary is lower $(s' < \hat{s})$ and P is lower $(\frac{\partial P}{\partial t} < 0)$. This contradicts that $(\hat{s}, \hat{b}, \hat{t})$ optimal.

■

Lemma 2 *If $t = e - z$ implements equilibrium effort for some z, then so does $t = e + z$.*

Proof of Lemma 2: Suppose (s, b, t) is an optimal contract generating equilibrium effort $e(s, b, t)$. Further suppose $t = e - z$ for some $z > 0$. Call this t_L. Let $t_H = e + z$. Then,

$$C'(e) = g(e - t_L)\Delta = g(z)\Delta = g(-z)\Delta = g(e - t_H)\Delta \tag{33}$$

The first inequality comes from (IC), the second from the definition of t_L, the third by symmetry[13] of g, and the fourth by the definition of t_H. Therefore, t_H also satisfies (IC). The same argument works if $z < 0$. ■

Proof of Proposition 3: For short hand, we use the following notation to ease exposition:

$$g = g(e - t) = g(t - e), \frac{\partial g}{\partial e} = g' = -\frac{g}{\partial t}, G = G(e - t), G(t - e) = 1 - G. \tag{34}$$

[13]This is an example of how symmetry eases the calculation of the targets. If the distribution is symmetric, the targets are equally spaced around equilibrium effort. Without symmetry, the targets would not be equidistant from the equilibrium effort, as long as the distribution is still single-peaked. If the distribution was not even single-peaked, there would be multiple targets.

The firm maximizes profits subject to (IC) and (PC) given by the program in (14). Write the Lagrangian as

$$L = e - (s + bG) + \lambda(u(s) + \Delta G) + \mu(g\Delta - C'(e))\,. \tag{35}$$

Recall that the chain rule gives

$$\frac{\partial \pi}{\partial k} = \Pi_e(e,k)\frac{\partial e}{\partial k} + \Pi_k(e,k) = 0\,. \tag{36}$$

Therefore we have the following partial derivatives:

$$\begin{aligned}
L_e &= 1 - bg + \lambda(\Delta g) + \mu(g'\Delta - Ce) & (37)\\
L_b &= -G + \lambda(u'(s+b)G) + \mu g u'(s+b) & (38)\\
L_s &= -1 + \lambda(u'(s) + (u'(s+b) - u'(s))G) + \mu g(u'(s+b) - u'(s)) & (39)\\
L_t &= bg + \lambda(\Delta(-g)) + \mu(\Delta(-g')). & (40)
\end{aligned}$$

Differentiate L with respect to b to get

$$[(1 - bg) + \lambda\Delta g + \mu(g'\Delta - c)]\frac{gu'(s+b)}{c - \Delta g'} - G + \lambda u'(s+b)G + \mu g u'(s+b) = 0\,. \tag{41}$$

Observe that $\mu(g'\Delta - c)$ cancels out by (IC):

$$(1 - bg)gu'(s+b) + \lambda[\Delta g^2 u'(s+b) + u'(s+b)G(c - \Delta g')] = G(c - \Delta g')\,. \tag{42}$$

Rearrange terms to arrive at

$$\frac{1}{u'(s+b)} = \lambda + \frac{(1 - bg)g + \lambda\Delta g^2}{GX}\,, \tag{43}$$

where $X = c - \Delta g' > 0$ by (SOC). Differentiate with respect to salary to get

$$[(1 - bg) + \lambda\Delta g + \mu(g'\Delta - c)]\frac{g\Delta}{c - \Delta g'} - 1 + \lambda[u(s) + \Delta' G] + \mu g\Delta' = 0\,. \tag{44}$$

Once again the terms $\mu(g'\Delta - c)$ cancel out, so

$$(1 - bg) + \lambda[\frac{\Delta g^2 \Delta'}{c - \Delta g'} + u'(s) + \Delta' G] = 1\,. \tag{45}$$

Differentiate L with respect to t to get

$$[(1 - bg) + \lambda\Delta g + \mu(g'\Delta - c)](-\frac{g'}{c}) + bg - \lambda\Delta g - \mu\Delta g'\,. \tag{46}$$

The second order condition from the incentive constraint satisfies

$$-g'(t-e)(u(s+b)-u(s))-C''(e)<0. \tag{SOC}$$

Assume (SOC) holds. By the standard results, equations (43), (45), (46) are necessary and sufficient for an optimal solution.[14]

Throughout, use the shorthand $\Delta = u(s+b) - u(s)$. Because costs are quadratic, we can rewrite (SOC) as

$$c + g'(t-e)\Delta > 0. \tag{47}$$

From Corollary 3, (SOC), and (27) we have:

$$\frac{\partial e}{\partial t} - 1 = \frac{-c}{c + g'(t-e)\Delta} < 0. \tag{48}$$

Differentiate expected utility with respect to the target:

$$u(s)g(t-e)\left(1 - \frac{\partial e}{\partial t}\right) + u(s+b)g(e-t)\left(\frac{\partial e}{\partial t} - 1\right). \tag{49}$$

Inserting (48) and rearranging gives,

$$\frac{\partial EU}{\partial t} = g(t-e)\left(\frac{\partial e}{\partial t} - 1\right)\Delta < 0, \tag{50}$$

where we have used (SOC) from (IC). Therefore, raising the target lowers the manager's payoff. Now differentiate expected utility with respect to the salary:

$$u'(s)g(t-e)\left[-\frac{\partial e}{\partial s}\right] + u'(s+b)g(e-t) + \left(\frac{\partial e}{\partial s}\right) - C'(e)\frac{\partial e}{\partial s}. \tag{51}$$

From (SOC) and (26), we have,

$$\frac{\partial EU}{\partial s} = \frac{\partial e}{\partial s}[g(e-t)\Delta' - C'(e)] > 0. \tag{52}$$

[14] Hemmer (2013) is critical of the standard First Order Approach (FOA) in models like this where risk is exogenous. The FOA pertains to the tactic of taking firs order conditions under general contracts of output ($s(q)$) rather than the restricted bonus/target class here. Since I assume (SOC) holds, differentiating the firm's objective function with respect to the contract parameters should guarantee an optimal solution. I make no claims that assuming MLRP or CDFC is sufficient to guarantee optimality, as the standard literature does.

Therefore, lowering the salary lowers expected utility of the manager. The function $e(s,b,t)$ is given by the manager's incentive constraint, which determines his effort with respect to (s,b,t). We now wish to show that $\hat{t} < e(\hat{s},\hat{b},\hat{t})$.

Suppose (s,b,t) is an optimal contract. We know the firm cannot implement first best, so $t \neq e$. First suppose $t = t_L = e - z$ for some $z > 0$. By Lemma 2, there exists a t_H that also satisfies the incentive constraint and implements the same effort level:

$$C'(e) = g(e - t_L)\Delta = g(e - t_H)\Delta, \tag{53}$$

where $t_H = e + z > e - z = t_L$. But this higher target lowers the manager's expected pay given (s,b):

$$EU(s,b,t_H) = u(s) + \Delta G(e - t_H) < u(s) + \Delta G(e - t_L) = EU(s,b,t_L) = \bar{u} \tag{54}$$

where the last equality holds since (PC) binds by Lemma 1. Thus (s,b,t_H) violates (PC) and cannot be optimal. Now suppose $t = t_H = e + z$ for some $z > 0$, for the given optimal contract (s,b,t), which we can call (s_H, b_H, t_H). Again by Lemma 2, there exists a t_L that implements the same effort. By the same logic, this low target raises the manager's expected pay, thus relaxing (PC). But then the firm could simultaneously lower the salary to $s_L < s_H$ and bonus to $b_L < b_H$ such that (PC) tightens and incentives are unchanged:

$$\Delta(s_L, b_L) = u(s_L + b_L) - u(s_L) = u(s_H + b_H) - u(s_H) = \Delta(s_H, b_H) \tag{55}$$

This implements effort with the lower target at lower cost. Therefore, (s_H, b_H, t_H) cannot be optimal. Thus $\hat{t} < \hat{e}$.

■

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