Team Incentives under Private Contracting

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Abstract

I model a moral hazard in teams problem in which the principal offers private contracts. Public contracts are common knowledge to all agents, but private contracts are known only between the principal and each individual agent. Public contracts can induce efficient outcomes, but are subject to collusion between the principal and any given agent. Private contracts by construction are immune to this collusion and generate contracts that are subject to a budget balance constraint. Thus, the budget balance constraint emerges endogenously, whereas most team incentive problems take this constraint as given.

1 Introduction

There's no question that teamwork is vital to modern organizations, as teams permeate all levels of the corporate hierarchy (boards, executive teams, partnerships, project teams). Much of the prior literature has modeled these teams as a collection of agents who receive individual or joint contracts on their output. Yet an underlying assumption is that the contracts are public, namely, that all team members know the contracts offered to the others. In practice, contracts are largely private, and agents rarely know the full details offered to other members of the team.[1] I explore the economic effects on teamwork of private contracts and find the surprising result that private contracts in equilibrium will satisfy budget balance, a constraint that the literature on team incentives has long taken as given.

Public contracting is the benchmark case that has occupied most of the prior literature on team production. The principal simultaneously makes offers that are public (known to all parties); this gives rise to the bonding contract of Holmström (1982), in which every agent becomes a residual claimant on the firm's output, and the principal charges each agent the efficient output level up front. Though this achieves efficiency, one chief problem of this bonding contract is that the principal has an incentive to reduce output, even though he captures total surplus. He effectively pays more than $1 in wages for every dollar in output, and therefore has an incentive to strike a private contract with any one agent to reduce effort. This private contract would be amenable to both parties; the agent is held to his reservation utility anyway, and the principal gains if output falls. The possibility of private contracts breaks the bonding contract.

This begs the question of what the equilibrium looks like under full private contracting. I propose a model of private contracts, in which the principal makes contracts to a team of agents, but each agent only knows of their own contract. Each agent therefore must speculate on the contract given to other agents. As the contract that any agent sees could be a signal of the contracts offered to other agents, this creates a complex signaling game of multiple equilibria. To narrow the scope of the problem, I follow

[1]For example, a private conversation with a retired Big Four audit partner has revealed that Ernst & Young currently uses a "closed" system, in which each partnership share is not disclosed across partners (private contract). At one point, Arthur Andersen had an "open" system (public contract), but now most of the Big Four auditors keep their partner distributions secret. And for good reason, as compensation can be a sensitive topic that reveals performance, which both the firm and the partner may prefer to keep between each other, not announced to all parties.

the literature to consider only passive beliefs: if the principal deviates from an agent's equilibrium contract, that agent believes that all other agents continue to face their equilibrium contracts.

Under this assumption, the participation constraint for each agent now changes, since each agent only knows of his own contract. The main result shows that under a certain class of production functions (additive separability in each agents' effort choice), every equilibrium in the private contracting game will satisfy budget balance, classically defined over the team of productive agents (not including the principal). In this sense, the ability to privately contract generates the budget balance constraint endogenously, which the large literature on team incentives takes as given.

The intuition behind this result fundamentally stems from externalities. Under team production, since payment is made on group output there is a positive externality from any agent's effort choice on other agents. Once any given agent has already committed to exert his efficient effort, he creates a positive externality on the second agent, who implicitly has already received some benefit from the first agent's work. At that point, the principal and second agent are both better off if the second agent shirks. Private contracts can thus break the public contracting equilibrium. Ultimately, the principal's payoff function is invariant to changes in output if and only if budget balance holds exactly. In that case, the principal will have no incentives to privately contract to either increase or decrease any individual agent's effort. The additive separability on production ensures that there are no interaction effects between different agents that would lead to further private contracts. This is fundamentally why budget balance emerges in equilibrium under private contracting and separable production.

In this sense, the literature has come full circle. The initial literature assumed budget balance as an exogenous constraint imposed on team production because it resembled a feature observed in some specific teams (like partnerships). But the constraint has real economic content, as apparent through the private contracting game. Budget balance kills the principal's incentives to privately contract because it makes its payoff function insensitive to changes in output.

Surprisingly, the emergence of budget balance can still occur even in a more general model of complementarity in production. General complementarity will introduce interaction effects between the different agents. Normalizing the complementarity factor to one, I show that budget balance holds in every equilibrium of the bilateral contracting game, even when the effort of different team members are perfect complements rather than perfect substitutes. This provides some modicum of reassurance that the results are not limited to a narrow class of production functions.

Much of the literature splits between team production (Holmström (1982), Huddart and Liang (2005), Legros and Matsushima (1991), Legros and Matthews (1993), Miller (1997), and Rasmusen (1987)) and cost sharing or cost allocation (Rajan (1992), Ray and Goldmanis (2012), and Baldenius et al. (2007)). The two problems are formally equivalent. Segal (1999), Segal and Whinston (2003), and McAfee and Schwartz (1994) establish some of the early framework for bilateral contracting, in particular the role of externalities. Segal and Whinston (2003) address private contracts in procurement and in inter-firm relationships, but not in the specific framework relevant for this paper.

I do not consider private contracts between the agents themselves, as Holmström and Milgrom (1990), Macho-Stadler and Perez-Castrillo (1993), Ramakrishnan and Thakor (1991), Itoh (1993), and Varian (1990). Those papers all require some kind of individual performance measure, or the ability for agents to observe action choices of other agents that the principal cannot, which does not fit the setting of this paper. But more importantly, the temptation to privately contract rests with the principal, not with each individual agent. Even though the principal captures total surplus at the efficient effort profile, he still has the temptation to reduce aggregate output. It is this temptation that leads to privately contracting with any individual agent in order to reduce output.

The literature on repeated games has relevance for my results as well, such as Abreu et al (1993). For example, Baker et al (1994) show that implicit incentives can serve as a complement to explicit incentives in a repeated game between a single principal and single agent. Che and Yoo (2001) show that the optimal incentive scheme uses low powered group incentives in a repeated game with a principal and a team. Both of these papers, like much of the repeated game literature, lean on the repetition of a static game to sustain equilibria that would vanish in a one shot game, using folk theorem style arguments. While I do not consider repeated games here, the notion of private contracting conceptually takes place in a dynamic setting. Indeed, the broad literature on repeated games in contract theory provides additional context for the more traditional explicit incentives considered here. Future research will provide more explicit characterizations of this dynamic interaction, and it will illustrate more explicitly how the possibility of building a reputation may make available equilibria that were previously unavailable in a one shot game.

2 The Model

Consider a risk-neutral principal contracting with n risk-neutral agents. Each agent i exerts effort $e_i \geq 0$ at cost $C_i(e_i)$, where $C_i' > 0, C_i'' > 0$ and $C_i(0) = 0$. Let $N = \{1..n\}$ be the team and let $e \equiv (e_i)_{i \in N}$ be the effort vector of all agents. Let $q : \mathbb{R}_+^n \to \mathbb{R}_+$ be the team production function.[2] Each agent observes only his own effort, while all parties (principal and all agents) observe joint output $q = q(e)$. Let $q_i(e) \equiv \frac{\partial q(e)}{\partial e_i} > 0$, and $\frac{\partial^2 q(e)}{\partial e_i^2} \leq 0$. Thus the team production function exhibits diminishing marginal returns and is increasing. Furthermore, I remain agnostic on the cross partial $\frac{\partial q_{ij}(e)}{\partial e_i \partial e_j}$, which could be positive, zero, or even negative. A positive cross-partial would indicate complementarity between agents, while a zero cross-partial would imply that their efforts are perfect substitutes.

The first best allocation e^* maximizes total surplus:

$$e^* \in \arg\max_e q(e) - \sum_i C_i(e_i), \tag{1}$$

yielding $q_i(e^*) = C_i'(e_i^*)$. The marginal cost of effort equals its marginal return, given by the marginal productivity of any given agent's effort on team production. First best output is $q^* \equiv q(e^*)$.

2.1 Public Contracting

A bilateral contract is a salary and bonus (s_i, b_i) for agent i where

$$w_i = w_i(q) = s_i + b_i q \tag{2}$$

is agent i's wage. I restrict attention to wages that are linear in output in order to gain traction on the sharing rules b_i of joint output q. Moreover, this fits many common applications in practice, in which partnerships receive a fixed salary (possibly negative, to allow a buy-in to the partnership). I do not impose limited liability on the contract, though in equilibrium the bonus terms will be positive. McAfee and MacMillan (1991) show that linear contracts on joint output are optimal in a model of moral hazard and

[2] There is no measurement error in output. Because joint output is pooled across agents, there is inherent difficulty in measuring the performance of each agent, even if output is measured without noise. Because agents are risk-neutral, adding a noise term is straightforward and simply involves replacing q with Eq throughout the paper. To ease analysis, I omit the noise term.

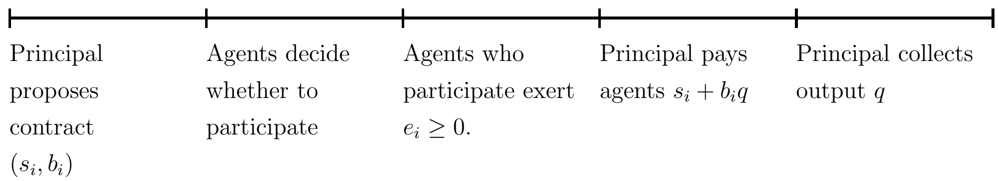

Figure 1: Timeline of the model.

adverse selection. As such, restricting attention to linear contracts may make sense if there is a larger underlying adverse selection problem.[3]

The principal *publicly* offers a bilateral contract (s_i, b_i) to each agent. Importantly, the principal can commit to this public offer, so he can commit not to privately contract with any agent. The game proceeds as follows: (1) the principal publicly makes bilateral contract offers (s_i, b_i); (2) each agent accepts or rejects the contract offered to him; (3) agents who accept the contract work; (4) the principal pays out wages w_i, and collects q. Observe that the agents work only if they participate. Should they decide to reject the contract, the principal does not capture their output. Figure 1 outlines the timeline of the model.

Let $s = (s_i)_{i \in N}$ and $b = (b_i)_{i \in N}$ be the salary and bonus vectors, respectively. An equilibrium is a tuple $\langle (\hat{s}, \hat{b}), \hat{e} \rangle$ such that each agent (best) responds to his offered contract $(\hat{s}_i, \hat{b}_i)$ and to the effort choices of all other agents, and the principal chooses the optimal (s, b) given that each agent is best responding. The principal offers the contract to all agents simultaneously, and then each agent chooses his effort in a non-cooperative game simultaneously with all other agents. As is common with team production problems, there is a large multiplicity of equilibria. To ease analysis, throughout I examine the principal's preferred Subgame Perfect Nash Equilibrium (SPNE) of this game. Effort $\hat{e}$ is an SPNE if each agent maximizes $w_i(q) - C_i(e_i)$ given $q = q(e_i, \hat{e}_{-i})$, or

$$\hat{e}_i = \arg\max_{e_i} s_i + b_i q(e_i, \hat{e}_{-i}) - C_i(e_i), \tag{3}$$

whose first order condition is the incentive constraint:

[3] Of course, I focus on the moral hazard issues to isolate the effects of private versus public contracting. Future work will layer an adverse selection problem on top of the moral hazard problem. Moreover, I focus on linear contracts given their widespread use in partnership agreements. Nonlinear contracts may improve on linear contracts.

$$b_i q_i(\hat{e}) = C_i'(\hat{e}_i) \tag{IC}$$

Observe the second order conditions hold by assumption:

$$b_i \frac{\partial q_i(\hat{e})}{\partial e_i} - C_i''(\hat{e}_i) < 0. \tag{SOSC}$$

Note that a bonus vector $b = (b_i)_{i \in N}$ induces an effort vector $e(b)$, so the principal's optimal $\hat{b}$ induces an optimal effort vector $\hat{e} \equiv e(\hat{b})$. Call a contract $(\hat{s}, \hat{b})$ an equilibrium contract if $\langle (\hat{s}, \hat{b}), e(\hat{b}) \rangle$ is an equilibrium.

Each agent has an outside option, normalized to zero. The principal can always offer each agent their outside option of $s_i = b_i = 0$, so $e_i = 0$. It is without loss of generality to restrict attention to equilibria in which all agents participate. Each agent will participate if he gets nonnegative utility in equilibrium, or $w_i - C_i(e_i) \geq 0$:

$$s_i + b_i q \geq C_i(e_i) \tag{PC_i}$$

The principal will select each s_i such that (PC_i) binds. As usual, this occurs because the salary only transfers rent between the principal and each agent, but does not affect the incentive constraint. The principal will substitute the binding (PC_i) into her payoffs $\pi = q - \sum_i w_i$ so:

$$\hat{b} \in \arg\max_b q(e(b)) - \sum_i C_i(e_i(b)). \tag{4}$$

The principal's profit function is identical to total surplus, and therefore the principal can implement efficient effort by choosing $\hat{b}_i = b_i^* = 1$, so $\hat{e}_i = e_i^*$. From binding (PC), optimal salary is

$$s_i^* = C_i(e^*{}_i) - q^* < 0, \tag{5}$$

since total surplus $= q^* - \sum_i C_i(e^*{}_i) > 0$. Thus, the principal can implement the first best under public contracts. This is the standard bonding contract, where the principal compensates each agent for his individual cost of effort, and each agent pays q^* up front. The principal makes each agent the residual claimant to guarantee efficient labor supply. Since the agents are held to their participation constraints, the principal earns all the rents, so the principal's profit is $\pi^* > 0$.[4] Every agent faces a negative salary,[5] similar to the feature of many professional partnerships that require partners to buy into the partnership, such as in accounting, law, or medicine. Moreover, the large tournaments literature pioneered by Lazear and Rosen (1979) use these bonding contracts: the agents are risk neutral, the win/loss prizes are large, and the principal collects the surplus from the agents through a bonding fee upfront.

The principal acts as an independent party who shares in the output of the team but bears no cost of production. As Holmstrom (1982) noted, introducing such a third party achieves efficiency even under "budget balance." Of course, the budget is balanced in a trivial sense, because the third party plays no role in production, and any choice of sharing rules would balance the budget because the third party acts as a sink (Miller 1997).[6] See Rasmusen (1987) for further analysis of the sink, such as the use of scapegoat and massacre contracts. A scapegoat contract punishes a single agent if the team deviates, and a massacre contract punishes all if a team deviates. Rasmusen (1987) examines team production under risk aversion, where such extreme contracts are welfare-enhancing. However, this is not relevant for this setting because of the risk neutrality of the agents. Both of these scapegoat and massacre contracts are nonlinear solutions that implement first best. They are multi-agent versions of the classic forcing contract in which the principal pays a prize if the agent achieves first best output, but nothing otherwise. These contracts, however, face criticism since they require full commitment from the principal. The principal will need to know the solution it seeks to induce and commit to levying harsh penalties if output varies from that solution. I focus on linear contracts for their wide practical use as well as lower commitment requirements.

Every agent earns the full return to their labor but pays for this through a large negative salary. In practice, limited liability constraints of agents will prohibit the use

[4]Note that the principal could easily break even by raising the salaries s_i to the point that $\pi = 0$. This passes the surplus from the principal to the agents, but does not change incentives. This would still be a SPNE. However, the principal prefers the SPNE in which she takes all the rents, and the agents are held to their reservation utilities.

[5]If agents faced a general outside option $\bar{u} > 0$, the efficient salary would be $\bar{u} + C_i(e_i^*) - q^*$. This salary could be positive for high enough outside options because the salary is a linear increasing function of $\bar{u}$. These outside options do not change the bonus contract, so the contract overall is qualitatively unchanged, making the normalization of $\bar{u} = 0$ without loss of generality.

[6]To see that the budget is balanced, let $w_p = q - \sum_i w_i$ be the principal's wage. Then the sum of all wages is trivially equal to output: $w_p + \sum_i w_i = q$.

of large negative salaries. But this is not the only problem with the efficient contract, as it leads to collusion between the principal and agent.

3 Private Contracting

In many contracting scenarios (such as employment), the principal contracts privately with each agent, even when the agents work together on a joint project. If the principal cannot commit to public contracts, but instead can make private contracts with each agent, the equilibrium contract above can no longer implement the first best effort allocation. To see this, consider a simple example.

Example: Let $n = 2$. Let output be $q(e_1, e_2) = e_1 + e_2$. Let $C_i(e_i) = \frac{1}{2}e_i^2$, so $C_i'(e_i) = e_i$. First best effort is $e_i^* = 1$, first best output is $q^* = 2$, and cost of effort is $C(e_i^*) = \frac{1}{2}$. Total surplus is $q^* - \sum C_i(e_i^*) = 1$. The efficient bonus that implements first best is $b_i^* = 1$ with a salary of $s_i^* = \frac{1}{2} - 2 = -\frac{3}{2}$. Each agent earns $u_i^* = 0$. The principal's profit is $\pi^* = q^*(1 - \sum b_i^*) - \sum s_i^* = 1$. Suppose that both agents agree to this contract. However, after signing the contract but before working, the principal makes a private contract with the second agent. He renegotiates by proposing a new contract $(\widetilde{s}_2, \widetilde{b}_2)$ where $\widetilde{s}_2 = \widetilde{b}_2 = 0$. Facing this contract, the agent will select effort $\widetilde{e}_2 = 0$ earning utility $\widetilde{u}_2 = 0$, making him indifferent between this new contract and his original one. If he accepts the new private contract, then the principal pays for output only from the first agent, so his profit is

$$\widetilde{\pi} = e_1^* - b_1^* e_1^* - s_1^* = \frac{3}{2} > 1 = \pi^* . \tag{6}$$

■

In the example, the principal has an incentive to privately contract with one agent in order to reduce his output. Even in this simple example of perfect substitutes and identical agents, the principal is better off by reducing the second agent's effort to 0. Indeed, this logic is general, and such private contracts break the equilibrium under public offers. More generally, the equilibrium under public contracts is no longer stable under private contracting:

Proposition 1 *If the principal cannot commit to public offers, then $b_i^* = 1$ is never an equilibrium. As a result, the first-best is not implementable.*

The principal is strictly better off by privately contracting with agent k to shirk. Intuitively, if the principal sets $b_i = 1$ for each i, she pays n dollars in wages for every dollar of output, so she has an incentive to privately contract with one agent to reduce his effort, yielding output $\tilde{q} < q^*$. She earns less revenue, but she also saves $C_k(e_k^*)$ from not employing agent k, and $(q^* - \tilde{q})$ on each of the $n-1$ workers who do work. The net benefit is positive if $n \geq 2$. Once I replace the public offers with private contracts, the original equilibrium is no longer stable. There are many other possible private contracts in this new game that break the equilibrium $(\hat{s}, \hat{b})$ in the original public offers game above.[7]

Proposition 1 complements Eswaran and Kotwal (1984), who find that if the principal cannot commit to public offers, group penalty schemes cannot implement first-best effort in a moral hazard in teams setup. Group penalty schemes are nonlinear forcing contracts, paying out bonuses to all agents only if all agents select first-best effort. My result gives a similar impossibility in the world of linear contracts on joint output. Both results rely on private contracting to break the public contracting equilibrium.

The extreme punishment contracts, such as the scapegoat and massacre contracts, will still suffer from the same effects of private contracting as the standard efficient bonding contract. In particular, these extreme punishment contracts can induce any effort level, say, e^*. Because of this, if the principal could privately contract with any other agent, he would do so after all other agents have committed to e_{-i}^*. The principal still has an incentive to reduce output for any agent. In other words, the extreme punishment contracts are not subgame perfect in the private contracting game.

3.1 Budget Balance

It turns out that there is a very close link, proven later, between private contracting and budget balance, the condition that the bonus coefficients across all agents sum to one.

[7] More generally, suppose the principal offers agent k an additional payment s_k to reduce output to some $\tilde{q} < q^*$. Before, the principal earned $\pi^* = q^* - \sum_i w_i^* = q^* - \sum_i (s_i + b_i q^*) = q^*(1-n) - \sum_i s_i$, whereas now she earns $\tilde{\pi} = \tilde{q}(1-n) - \sum_i s_i - s_k$. The principal gains if $\tilde{\pi} > \pi^*$, or if $s_k < (\tilde{q} - q^*)(n-1)$. The agent will agree if $w_k^* = s_i + q^* < \tilde{w_k} = s_i + \tilde{q} + s_k$, or if $s_k > (q^* - \tilde{q}) > 0$. Hence, both parties accept this new contract if $s_k \in (q^* - \tilde{q}, (q^* - \tilde{q})(n-1))$.

Definition 1 *A bonus vector b satisfies Budget Balance (BB) if* $\sum b_i = 1$.

This is a common and well known feature of most team incentive problems. It shows up in both team production as well as in the cost allocation literature.[8] Both of these literatures usually take budget balance as an exogenous constraint on the contracting problem. They largely defend the assumption on intuitive grounds, such as a cost allocation rule that is "tidy" as in Demski (1981).

Consider the bilateral surplus between the principal and some agent k:

$$\mathrm{BS}_k(e) = q - \sum_i w_i + w_k - C_k(e_k) = q(e) - \sum_{i \neq k} w_i(q(e)) - C_k(e_k). \tag{7}$$

Bilateral surplus is unlike total surplus because it does include some transfers. Recall that total surplus is a sum of all payoffs, both from the set of productive agents, as well as from the principal who collects output. As such, transfers between agents fall out of the surplus function. Since bilateral surplus only considers the joint utility of a principal and one agent, wages of all other agents will remain inside the surplus function. This is the summation on the right hand side of the equation above.

Definition 2 *Call a contract* $(\hat{s}, \hat{b})$ *bilaterally efficient if*

$$\hat{e}_k \in \arg\max_{e_k} BS_k(e_k, \hat{e}_{-k}) \tag{8}$$

for each k*, where* $\hat{e} = e(\hat{b})$ *is the effort generated by* $\hat{b}$.

In words, each agent's effort is generated by a bilaterally efficient contract that maximizes the bilateral surplus between the agent and the principal, holding all other agents to their contractually generated effort levels.

Corollary 1 *If* (s, b) *is bilaterally efficient, then budget balance holds.*

Corollary 1 shows that maximizing bilateral surplus yields a budget balancing contract. Indeed, the bilateral surplus is simply joint output less the wages of all other agents, less the cost of effort of agent k. However, when maximizing bilateral surplus,

[8]Specifically, Moulin (2008), Moulin and Shenker (1992), and Ray and Goldmanis (2012) impose budget balance in a cost allocation problem, and Huddart and Liang (2005), Legros and Matsushima (1991), Legros and Matthews (1993), and Miller (1997) impose budget balance in a partnership setting.

the firm can substitute in the incentive constraint for agent k, which includes that agent's bonus. Thus, the wage payments include the bonus of all other agents plus the bonus for agent k, which itself is the bonus of the entire team.

This must balance perfectly the coefficient on team output, which is one. This intuitively expresses why budget balance must hold. The proof of Corollary 1 provides a direct proof, but a simple argument provides the intuition. If the budget balance failed, then the sum of the bonuses may exceed or fall short of unity. Thus, there would be one agent k who would have a bonus strictly greater than or less than one. If the former, the principal would have an incentive to sign a contract with that agent to decrease his bonus, causing him to reduce effort and the principal to save on costly incentive payments. If the latter, the principal would have an incentive to sign a contract with that agent to increase his bonus, which would elicit more effort out of him and generate higher joint output. This would violate the initial claim that the original contract was bilaterally efficient, since the principal and agent k would now be better off.

To fix ideas, consider output functions that are additively separable, in which output is a linear combination of the effort of each agent.

Definition 3 *Output $q(e)$ is additively separable if $q(e) = \sum_{i=1}^{n} \alpha_i e_i$.*

Now, α_i represents the differential contribution of each agent to the team's output. Observe that α_i and effort are complements, so α_i is a productivity parameter for agent i. High α_i agents enjoy a higher return from their labor. To focus the analysis on moral hazard, I do not assume α_i to be private information but rather common knowledge to all participants.

It turns out that budget balance and separability determine the optimal bonuses.

Proposition 2 *Suppose q is additively separable and costs are quadratic (so $C_i(e_i) = \frac{c_i}{2}e_i^2$ for $c_i > 0$). If a contract maximizes total surplus subject to budget balance, then the unique bonus is given by:*

$$b_i = 1 - \frac{\frac{c_i}{\alpha_i^2}}{\frac{1}{n-1}\sum_i \frac{c_i}{\alpha_i{}^2}}. \tag{9}$$

As shown earlier, maximizing total surplus alone gives the unique contract of $b_i^* = 1$, which generates efficiency. Imposing budget balance constrains the bonuses to $\sum b_i = 1$, and their optimal level is given in the proposition. It is easy to verify that budget balance holds for (9).

3.2 General Analysis

The principal offers the contract (s_i, b_i) to each agent privately, so agents cannot observe each other's contracts. Each agent's decision to participate now depends on his beliefs about offers extended to the other agents. When the agent receives an offer from the principal he must speculate on the offers made to other agents. As such, the initial offer to agent i may serve as a signal on the offers to other agents. While arbitrary beliefs on out-of-equilibrium offers can generate many Perfect Bayesian equilibria, I will follow Segal (1999) and McAfee and Schwartz (1994) by restricting attention to "passive beliefs": after observing an (out-of-equilibrium) offer from the principal, an agent believes that the other agents continue to face their equilibrium offers. Call this game the private contracting game. The timeline of the private contracting game is identical to the public contracting game, as the only difference is the observability of the contracts.

Suppose $\hat{b}$ is the equilibrium bonus vector, generating equilibrium effort $\hat{e} = e(\hat{b})$. If the principal offers b_i to agent i, then the agent will accept the contract if he earns nonnegative utility, *given* that all other agents still choose their equilibrium effort $\hat{e}_{-i}$. The principal can always default to no trade and hold each agent to $s_i = b_i = 0$. Therefore the participation constraint is now

$$s_i + b_i q(e_i, \hat{e}_{-i}) \geq C_i(e_i) \qquad (PC_i')$$

This (PC_i') differs from (PC_i) in that the other agents' effort is held fixed at the equilibrium levels. This matters because the principal will substitute the participation constraint into the objective function, and whether e_{-i} is held at its equilibrium level or not will alter the principal's objective function. This is the only operational difference between public and private contracting. The rest of the problem (such as the incentive constraint) remains the same. Also, (PC_i') is a more restrictive condition than the participation constraint under public contracting, since here e_{-i} are constrained at their equilibrium levels. So the principal is (weakly) worse off under private contracting.

Regarding the assumption of passive beliefs, Segal and Whinston (2003) examine equilibria without imposing exogenous restrictions on the agents' beliefs. Their principal offers a menu of contracts instead of a point contract, and this places a bound on equilibrium outcomes in a class of bilateral contracting games. Unfortunately, that class of games does not include the model here. Moreover, these other games do not present a single outstanding alternative to the passive beliefs assumption. Indeed, the beauty of the passive beliefs assumption is that it is completely consistent with Nash equilibria because it allows all other agents to hold their effort at their equilibrium levels, just like the standard Nash assumption. Alternative belief structures would need to specify exactly what the other agents will do. One extreme version is pessimistic beliefs: If agent i receives a private offer, he assumes that all other agents have received offers to shirk; therefore, they will exert effort $e_j = 0$ for $j \neq i$. This would change the participation constraint from (PC_i') to $s_i + b_i q(e_i, 0) \geq C_i(e_i)$. As is common with Perfect Bayesian equilibria, the analysis is only unique up to the specification of the beliefs, which can be arbitrary. In particular, it is possible that the agents hold beliefs that all other agents will receive any conceivable mix of offers. Ultimately, the most straightforward belief system, passive beliefs, is largely consistent with equilibrium and tractable, which is why I follow the literature by adopting it here.

By the usual arguments, the principal will select the salary such that the participation constraint binds. So, the principal's objective function is to maximize

$$V = q(e) - \sum_{i=1}^{n} \Big[C_i(e_i) + b_i \Big(q(e) - q(e_i, \hat{e}_{-i}) \Big) \Big] . \tag{10}$$

Externalities will be crucial to the private contracting game. Observe that the last term $E_i \equiv q(e) - q(e_i, \hat{e}_{-i})$ is the externality imposed on agent i when all other agents select an effort vector e_{-i} different from their equilibrium effort vector $\hat{e}_{-i}$. Indeed, this is the externality of all other agents on agent i. If this externality was 0, then this term E_i would vanish and the principal would maximize total surplus, leading to efficiency. But the presence of this externality will constrain the principal's problem to choose a bonus vector that will not implement first best.

Furthermore, in the choice of bonus coefficient there is a direct effect and an indirect effect. The direct effect on the principal's profit is the cost of the bonus payment that the principal makes to each agent, and the indirect effect is the effect of the bonus on the effort of each agent. Under arbitrary production functions, a bonus for agent i may affect the effort for some agent $j \neq i$, which may lead to many interaction terms. However, the direct effect is more simple. Observe that $\frac{\partial V}{\partial b_i} = E_i$, so the change in the principal's payoff from a marginal change in bonus is exactly this externality. This effect vanishes in equilibrium, since by construction $\hat{E}_i = q(\hat{e}) - q(\hat{e}_i, \hat{e}_{-i}) = 0$. This leaves only the indirect effect of the bonus on effort. This is ultimately an envelope theorem argument.

Proposition 3 *If q is additively separable, the principal's preferred equilibrium of the private contracting game satisfies budget balance.*

This makes the budget balance constraint endogenous. The intuition stems from writing the principal's payoff as:

$$\pi = q - \sum w_i = q\left(1 - \sum b_i\right) - \sum s_i\,. \tag{11}$$

Observe that the coefficient on output, marginal profit $\pi'(q) = (1 - \sum b_i)$ can be positive, negative, or zero. Under the efficient contract, this coefficient is negative, and so the principal is actually worse off for marginal increases in output. Every marginal increase in output drains his profits because of the high cost of securing that output. As such, the principal will have an incentive to privately contract in order to decrease output. Similarly, if the coefficient $(1-\sum b_i)$ is positive, then the principal would enjoy a marginal increase in profit for every increase in output. So the principal would have an incentive to privately contract with an agent to increase his output to the team. Intuitively, the principal will do this up to the point at which the coefficient is 0. At that point, the bonus perfectly sums to unity ($\sum b_i = 1$), and a marginal change in output leads to no change in profit for the principal; this kills the principal's incentives to privately contract. This is ultimately the reason why budget balance emerges endogenously. In order to avoid privately contracting the principal's payoff must be invariant to marginal changes in output. This only occurs when budget balance holds.

Observe that Proposition 3 holds for any vector of weights (α_i) across different agents. Regardless of how productive one agent is against another, budget balance will always hold. Importantly, the weights across different agents do not alter total incentives. Formally, the proof of Proposition 3 shows that the key analytical feature that generates budget balance is the behavior of $E_i = q(e_i, e_{-i}) - q(e_i, \hat{e}_{-i})$, the externality imposed on an agent by changing the other agent's effort levels. In particular, under additively separable production, this term is independent of any given agent's effort. Said differently, $q(e_i, e_{-i}) - q(e_i, \hat{e}_{-i})$ does not depend on e_i.

Example: Continuing the prior example, where $n = 2$, output is $q(e) = e_1 + e_2$ and cost of effort is quadratic, so $C'(e_i) = e_i$. The principal selects salaries such that (PC_i') binds. Here, $E_i = \sum_{j\neq i}^{2}(e_j - \hat{e}_j)$. In this specification, his payoffs are

$$q(e) - \sum_{i=1}^{2} \left[\frac{1}{2} {e_i}^2 + b_i \sum_{j \neq i} (e_j - \hat{e}_j) \right]. \tag{12}$$

Writing this out for $n = 2$ gives

$$(e_1 + e_2) - \left(\frac{{e_1}^2}{2} + \frac{{e_2}^2}{2} \right) - b_1(e_2 - \hat{e}_2) - b_2(e_1 - \hat{e}_1) \tag{13}$$

Taking first order conditions and rearranging gives

$$(1 - e_1) = b_2 + (e_2 - \hat{e}_2) \tag{14}$$

$$(1 - e_2) = b_1 + (e_1 - \hat{e}_1) \tag{15}$$

In equilibrium, $e_i = \hat{e}_i$. Looking within this symmetric equilibrium, I know $e_i = e_j$. From the incentive constraint, $b_i = e_i$ since the marginal cost of effort is 1. Therefore, $b_i = \frac{1}{2}$ is the unique solution. Thus, every agent receives $\frac{1}{2}$ of the total surplus. And the equilibrium effort is $\hat{e}_i = \frac{1}{2} < 1 = e^*$, distorted below first best. ■

The solution to Proposition 3 under $\alpha_i = 1$ and quadratic costs $C_i(e_i) = \frac{c_i}{2} {e_i}^2$ is

$$\hat{b}_i = 1 - \frac{c_i}{\frac{1}{n-1} \sum_j c_j}. \tag{16}$$

The principal reduces the bonus of each agent according to the ratio of his cost of effort to the average cost of effort of the other agents. An agent with a high cost of effort relative to his peers receives a small incentive payment. Hence, the principal grants a larger share of output to the agents with less elastic labor supply. From (IC), agents work harder with greater incentives, and so the agents with the least costly effort work the hardest on the team.

One consequence of separability is that it gives a precise direction on the effort distortion.

Corollary 2 *If $\alpha_i = 1$ and q is additively separable, efficient effort exceeds equilibrium effort, in every equilibrium. Specifically $\hat{e}_i \leq e_i^*$, with equality for at most one agent.*

The intuition is straightforward. Budget balance constrains the bonuses to sum to one. Therefore, all agents must share in the total output of the firm. Alternatively, efficiency requires the principal to sell the firm to each agent individually, making each agent the full residual claimant and taking back the rents through the salaries. This will lead to an effort distortion since constraining the bonuses to sum to one must make each bonus (weakly) smaller than 1, and these diminished incentives reduce effort through the incentive constraint.

The unique equilibrium of the private contracting game aligns with budget balance and surplus maximization under separability.

Corollary 3 *Let q be additively separable and agents identical. A contract maximizes total surplus subject to budget balance if and only if it is the principal's preferred equilibrium of the private contracting game.*

If the incentives sum to 1, for each dollar of output, the principal pays at most 1 dollar in wages. Instead of selling the firm to each agent, the principal sells the firm to all agents collectively. If the principal maximizes total surplus subject to the budget balance constraint, then the resulting contract is an equilibrium in the private contracting game, and hence the principal has no incentive to collude (privately contract) with any agent off of equilibrium. It is certainly *not* the case that the principal is better off by imposing the constraint; in fact, the principal is worse off under private, rather than public contracting, since first best requires $b_i = 1$. Instead, the principal imposes the constraint because it delivers a contract that survives in the private contracting game. Not imposing the constraint generates contracts that the principal may certainly prefer, but such contracts are not equilibrium contracts. Private contracting is the most relevant form of contracting in employment, and this makes budget balance constraint relevant in a world of private contracting.

3.3 Discussion and Connection to Existing Literature

It is worthwhile to compare my results with those of Segal (1999), the chief paper that has established the theoretical groundwork on contracting with externalities. Broadly speaking, my model both is, and is not, an extension of Segal (1999). To see this, please see Figure 2, which lists both models in a side-by-side comparison.

Segal (1999) examines a class of contracting models with quasi-linear utility. While this class is broad, it does not perfectly capture the team incentive framework. A group of n agents contracts over outcome $x = (x_i)$ with transfers t_i to agent i. Each agent earns

	Segal (1999)	This Model
Agent Payoff	$u_i(x) - t_i$	$s_i + b_i q(e) - C_i(e_i)$
Principal Payoff	$f(x) + \sum_i t_i$	$q(e)(1 - \sum_i b_i) - \sum_i s_i$
Total Surplus	$f(x) + \sum_i u_i(x)$	$q(e) - \sum_i C_i(e_i)$
(PC_i)	$u_i(x) - t_i \geq u_i(0, x_{-i})$	$s_i + b_i q(e) - C_i(e_i) \geq 0$
Π_{Pub}	$f(x) + \sum_i [u_i(x) - u_i(0, x_{-i})]$	$q(e) - \sum_i C_i(e_i)$
(PC_i')	$u_i(x_i, \hat{x}_{-i}) - t_i \geq u_i(0, \hat{x}_{-i})$	$s_i + b_i q(e_i, \hat{e}_{-i}) - C_i(e_i) \geq 0$
Π_{Pvt}	$f(x) + \sum_i u_i(x_i, \hat{x}_{-i})$	$q(e) - \sum_i [C_i(e_i) + b_i(q(e) - q(e_i, \hat{e}_{-i})]$

Figure 2

$u_i(x) - t_i$ while the principal collects $f(x) + \sum t_i$, yielding total surplus $f(x) + \sum u_i(x)$. In contrast, see the right hand side of Figure 2 for the payoffs of each agent, the principal, and the total surplus in this paper.

The key difference at the outset is that the transfer in Segal (1999) is a single variable t_i, whereas the contract here is a pair (s_i, b_i). Thus, even though the salary acts as a transfer (with $s_i = -t_i$), the bonus on output is an additional variable that is not included in Segal (1999). The bonus term affects the marginal payoff for the principal and is absent from the principal's payoff in Segal (1999).

The linear contract of my model also affects the nature of the externalities. To see this, observe that the participation constraint under public contracting is given by (PC_i) in Figure 2. For Segal (1999), the right-hand side of the participation constraint is $u_i(0, x_{-i})$, which is the payoff to any given agent i if he chooses not to participate, but all other agents do. This term captures what Segal (1999) calls "externalities on non-traders," which are the non-participating agents who choose not to trade (or $x_i = 0$). In contrast, because each agent in my model earns a payoff of $s_i + b_i q$, if he chooses not to participate, he earns nothing ($s_i = b_i = 0$) and also exerts no effort. As such, the right-hand side of my participation constraint is zero. In other words, there are no externalities on non-traders here. This affects the principal's profit function under public contracts, expressed as Π_{Pub} in Figure 2. For Segal (1999), the residual term $u_i(0, x_{-i})$ constrains the principal's profit function and will distort the optimal choice away from efficiency. Since externalities on non-traders are absent here, the principal's payoff under public contracts is equal to total surplus. This is why I can obtain efficiency under public contracting, as the linear contract eliminates externalities on non-traders, and therefore public contracting maximizes total surplus.

Finally, the two models are more similar under private contracting. Observe the participation constraint (PC_i') under private contracting, shown in Figure 2. For Segal (1999), the right-hand side is $u_i(0, \hat{x}_{-i})$: the payoff to an agent who does not participate, even though all other agents continue to trade at their equilibrium levels $\hat{x}_{-i}$. The principal's profit under private contracting in both Segal (1999) and here includes the equilibrium term $\hat{x}_{-i}$ for Segal (1999) and $\hat{e}_{-i}$ for my model, and this in general will distort choices away from efficiency. The principal's payoff under private contracting here includes the externality term $q(e) - q(e_i, \hat{e}_{-i})$, whereas this is absent from the corresponding payoff from Segal (1999).

Segal (1999) also gives general conditions comparing public and private contracting. Let's consider these in turn. First, he shows that if externalities are absent at an efficient trade profile, private contracting produces efficient outcomes, regardless of externalities that may exist at other trade profiles. This result is also true here and in fact has a similar simple proof. If there exists an e^* such that $q(e_i^*, e_{-i})$ does not depend on e_{-i} for all i, then observe that for any equilibrium of the private game, the equilibrium condition reduces to

$$q(\hat{e}) - \sum C_i(\hat{e}_i) \geq q(e^*) - \sum \left[C_i(e_i) + b_i \left\{ q(e^*) - q(e_i^*, e_{-i}) \right\} \right] = q(e^*) - \sum C_i(e_i^*) \quad (17)$$

and therefore this equilibrium is efficient. Of course, this assumption is quite strong, in that the production function cannot depend on the actions of other agents, which is unlikely to hold in realistic settings. So in this case, our model fits that of Segal (1999).

However, Segal (1999) further shows that if the agent's utility function is additively separable in x_i and x_{-i}, then public and private contracting coincide. The corresponding assumption here is additive separability in the production function, which Proposition 3 and Corollary 2 has already shown makes private contracting differ from efficiency. And because public contracting is identical to efficiency, this means that private and public contracting in this model will differ.

Indeed, it is somewhat curious that the exact same assumption, additive separability, generates precisely opposite conclusions in Segal (1999) and here. Yet on inspection, this is not surprising. Because there are no externalities for non-traders (non-participating agents), public contracting and efficiency are identical. So, the bilateral public offers here generate efficient outcomes. The private contract generates budget balance, which Proposition 1 shows introduces inefficiency. Thus, the linear wage schedule forces a difference between public and private contracts. Segal (1999), on the other hand, does involve externalities on non-participating agents, causing public contracts to deviate from efficiency, and occasionally can actually coincide with private contracting.

An open question is how my results would change under more general, nonlinear contracts. Overall, precise results are difficult to obtain without specifying the functional form of the nonlinearity. I am able to obtain some fairly general results which are similar in spirit to Segal (1999).[9]

4 Complementarity in Production

Now consider the analysis under complementarity, in which one agent's effort can explicitly affect another agent's equilibrium effort choice. In general, this problem becomes complex, since complementarity introduces many interaction terms between agents. To fix ideas, consider the general case of full complementarity.

Definition 4 *The production function q has full complementarity if $q(e) = \prod_{i=1}^{n} \beta_i e_i$.*

The parameter β_i tracks the strength of the complementarity between agents. As each β_i grows, the effect of agent i on agent j increases. Throughout, let $\beta = \prod_i \beta_i$ be the product of these complementarity weights.

Observe that the marginal effect of any given agent's effort on joint output is given by $q_i(e) = \beta_i \prod_{j \neq i} \beta_j e_j$. Therefore all other agents $j \neq i$ will affect any given agent's impact on joint output. This extreme case of complementarity binds all agents together. Indeed, the incentive constraint in this case collapses to

$$C'(e_i) = b_i \beta_i \prod_{j \neq i} \beta_j e_j \,. \tag{18}$$

[9] Details of this nonlinear analysis are available from the author upon request.

In this specific setting with arbitrary number of agents, the number of possible interaction effects precludes closed form solutions. However, the case with $n = 3$ is remarkably elegant and generates a precise solution.

Proposition 4 *If q has full complementarity, under three identical agents and quadratic costs, the equilibrium is*

$$\hat{b} = \frac{\beta}{\sqrt{\beta} + 2\beta} \text{ and } \hat{e} = \frac{\sqrt{\beta} + 2\beta}{\beta\sqrt{\beta}}. \tag{19}$$

Equilibrium effort is distorted above first-best ($\hat{e} > e^*$). If $\beta = 1$, then budget balance holds in the equilibrium of the bilateral private contracting game.

Three agents allow a closed form expression for equilibrium effort as a function of the primitives of the model, namely, the complementarity parameters β_i and the bonus coefficient b. The expression for each agent's effort is $e = \left(\beta\sqrt{\prod_{j\neq i} b_j}\right)^{-1}$. It is easy to see that each agent's bonus has no effect on his effort level ($\frac{\partial e_i}{\partial b_i} = 0$), while the bonus of other agent's $j \neq i$ has a negative effect on his effort level ($\frac{\partial e_i}{\partial b_j} < 0$). This is precisely the consequence of complementarity: when the principal raises the bonus on other agents, this induces agent i to shirk, since the higher bonus now allows the agent to gain benefit without exerting costly effort.

As illustrated before, the principal's objective function is the sum of total surplus and the sum of the externalities of all other agents on each agent. Solving this optimization under full complementarity and three agents leads to several simplifications. First, $E_i = 0$ and $\frac{\partial E_i}{\partial b_i} = 0$ for <u>any</u> production function at equilibrium. More importantly, the null effect of the bonus on any individual agent effort ($\frac{\partial e_i}{\partial b_i} = 0$) markedly simplifies the first order conditions. Furthermore, under identical agents the first order condition provides a unique solution for effort $\left(e = \frac{1}{b\sqrt{\beta}}\right)$ as a function of the bonus coefficient b and β, the product of the β_i. Combining this with the incentive constraint generates a unique solution for bonus and effort as a function of β, given in the statement of proposition. The proof shows that the bonus increases in β while the equilibrium effort decreases in β. This should be intuitive: as the complementarity between different agents increases, their productivity grows and their value to the firm increases, so the principal increases the bonus in response. However, because of this complementarity, the total effect on effort will fall since increasing any other individual's bonus decreases that individual's effort ($\frac{\partial e_i}{\partial b_j} < 0$).

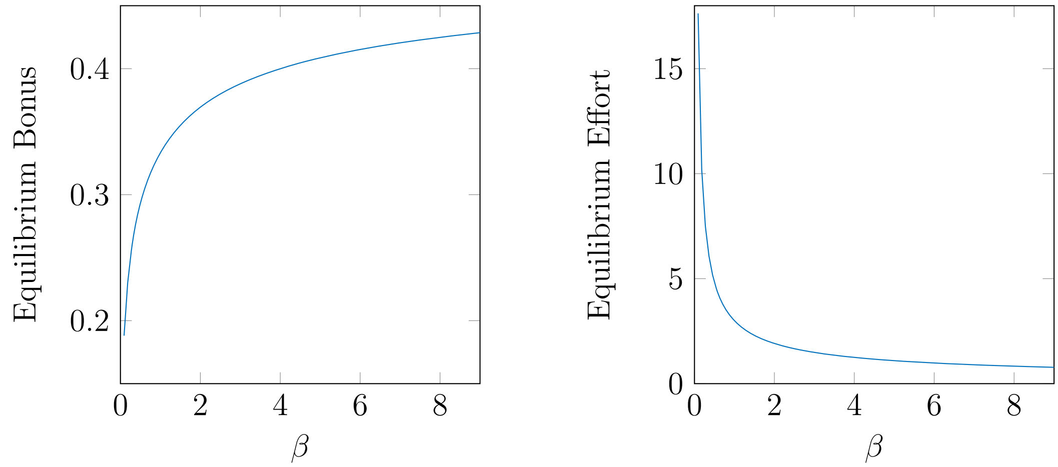

Figure 3: Equilibrium bonus and effort as a function of the complementarity factor β.

The complementarity factor β modulates the strength of interactions between agents. High β increases the synergies and, therefore, increases the marginal effect any one agent has on the others. In particular, if $\beta \neq 1$, then the budget balance will fail. The direction of the imbalance will depend on whether $\beta > 1$ or $\beta < 1$. Figure 3 plots the equilibrium bonus and effort as a function of this complementarity factor β. As β increases, so does the optimal bonus, while equilibrium effort decreases. To see this more explicitly, consider the following pair of examples.

Example: Let $n = 3$, $q(e) = \beta e_1 e_2 e_3$. Let $C_i'(e_i) = e_i$. Total surplus is $q(e) - \sum_i C_i(e_i)$, and therefore, $\beta e_j e_k = q_i(e) = C_i'(e_i) = e_i$ for $i \neq j, k$. Efficient effort under identical agents is $e^* = \frac{1}{\beta}$.

Suppose $\beta = 0.25$. From Proposition 4, $\hat{\beta} = \frac{1}{4}$ and $\hat{e} = 8$. Thus, the bonus falls short of budget balance (since $3\hat{b} = \frac{3}{4} < 1$) and equilibrium effort exceeds first best effort, as $\hat{e} = 8 > 4 = e^*$. Finally, the marginal bilateral surplus is given by $BS_k'(e) = q_k(e)\,[1 - \sum_i b_i] > 0$. This is positive since $q_k(e) > 0$, therefore, increasing the bonus to each agent increases bilateral surplus between the principal and any agent.

Alternatively, suppose $\beta = 4$. From Proposition 4, the equilibrium bonus is $\hat{b} = 0.4$ and the equilibrium effort is $\hat{e} = \frac{5}{4} > \frac{1}{4} = e^*$. Therefore, the bonus exceeds the budget balance. Equilibrium effort also exceeds the first best effort. The marginal bilateral surplus, given this contract, is $BS_k'(e) = q_k(e)\,[1 - \sum_i b_i] < 0$. Because $q_k(e) > 0$ and $\sum_i b_i = 3\hat{b} = 1.2 > 1$, the term is negative. Therefore, the principal and any agent are better off re-contracting to a lower bonus, since this will increase the bilateral surplus. ■

Observe the following features of the prior example. First, if the complementarity factor is low (with $\beta = \frac{1}{4}$), then the optimal bonus falls short of budget balance. In this case, the contract is not bilaterally efficient since the principal can increase bilateral surplus by increasing the bonus. This would benefit the principal and any one agent, and therefore, would make the pair better off. This is why the equilibrium of the private contracting game is not bilaterally efficient, and illustrates the difference between Proposition 4 and Corollary 1.

Similarly, with a high complementarity factor (of $\beta = 4$), the bonus exceeds budget balance. It is still the case that equilibrium effort exceeds first best effort. This shows that the effort distortion is a feature of the production function rather than the complementarity factor β. At this contract, the marginal bilateral surplus is negative, so the principal and any one agent is jointly better off with a lower bonus.

5 Conclusion

The problem of free riding has plagued teams ever since the dawn of time. But it was Holmström (1982) who first cast the fundamental tension between efficiency and budget balance: the requirement that the sum of output-contingent pay granted to individual team members equal total aggregate output, a common feature of partnerships. Nonetheless, Holmström proposed a solution of a "budget breaker," in which a third party (the principal) acts as a non-productive member of the team who coordinates contracting, finances the bonuses, and acts as a sink on the output of all agents. By including a budget breaker, the team achieves efficient production and still satisfies budget balance over the team that now includes the principal. Yet this solution has both conceptual and practical difficulties. The transfer payments from the agents violate endowment (or limited liability) constraints. But more importantly, the budget breaker solution falls apart if the principal can contract privately with the agents. This paper explores the consequences to team production under two alternative contracting regimes, public and private contracting.

The key insight of this analysis is that private contracting prevents collusion between the principal and the agent. This collusion is exactly what breaks the public contracting equilibrium. The only contracts that survive these rounds of private contracts are budget balancing contracts, providing a justification for the budget balance constraint which the team incentive literature has largely taken as given.

Future research in this area will explore more fully the consequences of private contracting, which characterizes most employment settings. For example, any number of contracting environments (such as corporate boards contracting with executives, managers contracting with their superiors, or contracts between firms in the supply chain) are all open to further analysis when the contracts are private. This just scratches the surface of a new literature which could more closely resemble the information environments faced in practice.

6 Appendix

Proof of Proposition 1: Let $(\hat{s}, \hat{b})$ be the equilibrium contract under public offers implementing the first best. So the bonus $\hat{b_i} = 1$ generates efficient effort, $e_i^* = e_i(\hat{b_i})$ where $C_i'(e_i^*) = 1$. And $\hat{s_i}$ satisfies the participation constraints, so each agent gets $u_i^* = \hat{s}_i + q^* - C_i(e_i^*) = 0$, or $\hat{s_i} = C_i(e_i^*) - q^* < 0$. The principal gets $\pi^* = q^* - \sum_i(\hat{s}_i + q^*) = -\sum_i \hat{s}_i + q^*(1-n)$. The principal collects revenue up front from the negative salaries, earns q^* in revenue from the output, and pays out q^* to each of the n agents.

Now suppose that after publicly offering $(\hat{s}, \hat{b})$, the principal privately offers to replace some agent k's contract with $\tilde{s_k} = \tilde{b_k} = 0$, causing him to shirk completely. Other agents know nothing of this contract, so they select the same effort level as before, since their own contract is unchanged. These agents will still choose to participate because they believe equilibrium output is still q^*. So $\tilde{u}_i = u_i^* = 0$ for all $i \neq k$ and $\tilde{u_k} = \tilde{s_k} = 0 = u_k^*$, and all agents are indifferent under the new contract. But the principal now earns

$$\tilde{\pi} = \tilde{q} - \sum_{i \neq k}(\hat{s}_i + \tilde{q}) = -\sum_{i \neq k} \hat{s}_i - \tilde{q}(2-n), \tag{20}$$

where $\tilde{q} = q(0, e_{-i}^*)$ is total output under the new contract. Recall that the salaries $\hat{s}_i < 0$ are a source of revenue, so paying agent k zero salary reduces revenue for the principal. But now she pays $\tilde{q}$ to $n-1$ agents instead of q^* to n agents, since agent k no longer works. So her wage costs fall as well. Since $\hat{s_k} = C_k(e_k^*) - q^*$ and $\tilde{q} < q^*$,

$$\tilde{\pi} - \pi^* = \hat{s_k} + \tilde{q}(2-n) - q^*(1-n) = C_k(e_k^*) + (q^* - \tilde{q})(n-2) > 0 \tag{21}$$

if $n \geq 2$. So the fall in wage costs exceeds the fall in revenue, and the principal is strictly better off.

■

Proof of Corollary 1: From (7), the bilateral surplus between the principal and some agent k is:

$$BS_k(e) = q(e) - \sum_{i \neq k} w_i(q(e)) - C_k(e_k). \tag{22}$$

Under linear contracts, write this as

$$BS_k(e) = q(e)\left(1 - \sum_{i \neq k} b_i\right) - \sum_{i \neq k} s_i - C_k(e_k). \tag{23}$$

Maximizing with respect to e_k gives the FOC

$$q_k(e)\left[1 - \sum_{i \neq k} b_i\right] - C_k'(e_k) = 0. \tag{24}$$

Substituting in the incentive constraint $b_k q_k(e) = C_k'(e_k)$ and collecting terms,

$$q_k(e)\left[1 - \sum_{i=1}^{n} b_i\right] = 0. \tag{25}$$

Since $q_k(e) > 0$, $\sum b_i = 1$. ■

Proof of Proposition 2: Suppose that the contract (s, b) maximizes total surplus subject to budget balance. Then the principal solves

$$\max_{b_i} q(e) - \sum\nolimits_j C_j(e_j) - \lambda(\sum b_j - 1) \tag{26}$$

Differentiating with respect to the bonus term b_i gives

$$\sum\nolimits_j (1 - b_j) q_j(e) \frac{\partial e_j}{\partial b_i} = \lambda \tag{27}$$

Recall that under quadratic costs $C_i'(e_i) = c_i e_i$. Under additive separability $q_j(e) = \alpha_j$, so the incentive constraint reduces to $b_i \alpha_i = c_i e_i$. Finally, each agent's bonus only affects his own effort, so

$$\frac{\partial e_i}{\partial b_i} = \frac{\alpha_i}{c_i} \text{ and } \frac{\partial e_i}{\partial b_j} = 0 \text{ for } j \neq i \tag{28}$$

Therefore, I can write the first order condition as:

$$(1-b_i)\frac{{\alpha_i}^2}{c_i} = \lambda. \tag{29}$$

Rearranging this gives $1-\frac{\lambda c_i}{\alpha_i^2} = b_i$. Summing both terms over all agents and imposing budget balance gives

$$\sum_i (1-\frac{\lambda c_i}{\alpha_i^2}) = \sum_i b_i = 1. \tag{30}$$

Simplifying this expression generates $\lambda = \frac{n-1}{\sum_i \frac{c_i}{{\alpha_i}^2}}$. Plug into (29) to get b_i. ■

Proof of Proposition 3: The principal solves

$$\hat{b} \in \arg\max_b q(e) - \sum_i (s_i + b_i q(e)) \text{ subject to } (PC'). \tag{31}$$

The principal will choose $\hat{s}_i$ such that (PC') binds, and substituting this into the profit optimization shows that

$$\hat{b} \in \arg\max_b q(e) - \sum_i \Big(C_i(e_i) + b_i q(e) - b_i q(e_i, \hat{e}_{-i}) \Big). \tag{32}$$

Now,

$$q(e, \hat{e}_{-i}) = \alpha_i e_i + \sum_{j \neq i} \alpha_j \hat{e}_j \Longrightarrow q(e) - q(e_i, \hat{e}_{-i}) = \sum_{j \neq i} \alpha_j [e_j - \hat{e}_j]. \tag{33}$$

Plug this into (32) to get

$$\hat{b} \in \arg\max_b q(e) - \sum_i \left(C_i(e_i) + b_i \sum_{j \neq i} \alpha_j (e_j - \hat{e}_j) \right) \tag{34}$$

Now, observe that

$$\sum_i b_i \sum_{j \neq i} \alpha_j (e_j - \hat{e}_j) = \sum_i \alpha_i (e_i - \hat{e}_i) \sum_{j \neq i} b_j \tag{35}$$

By additive separability,

$$\frac{\partial q(e)}{\partial e_i} = q_i(e) = q_i(e_i) = \alpha_i \Rightarrow \frac{\partial e_i}{\partial b_j} = 0 \text{ for } i \neq j.$$

So the derivative of this objective function (34) with respect to b_k is

$$\frac{\partial \pi}{\partial e_k}\frac{\partial e_k}{\partial b_k} + \frac{\partial \pi}{\partial b_k} = \left[\alpha_k - C_k'(e_k) - \alpha_k \sum_{i \neq k} b_i\right] \frac{\partial e_k}{\partial b_k} - \sum_{j \neq k} \alpha_j (e_j - \hat{e}_j). \tag{36}$$

The FOC requires this derivative evaluated at the equilibrium $\hat{b}$, to equal zero. So $\hat{e}_j = e_j(\hat{b}_j)$ and the last sum on the right vanishes, so

$$\left[\alpha_k - C_k'(\hat{e}_k) - \alpha_k \sum_{i \neq k} \widehat{b_i}\right] \frac{\partial e_k}{\partial b_k} = 0. \tag{37}$$

Combining with (IC) gives $\sum_i \widehat{b_i} = 1$.

■

Proof of Corollary 2: By additive separability, $q_i(e) = 1$. From Proposition 3, the equilibrium contract in the private contracting game satisfies budget balance, so $\sum b_i = 1$. There are two cases. First, suppose $b_i < 1$ for each agent. Then from the incentive constraint I have

$$C'(\hat{e}_i) = b_i < 1 = C'(e_i^*), \tag{38}$$

so $\hat{e}_i < e_i^*$ for each agent. Second, suppose there exists some agent j such that $b_j = 1$. Then from budget balance, it must be that $b_i = 0$ for all other agents $i \neq j$. From the incentive constraint, I have

$$C'(\hat{e}_i) = b_i = 0 < 1 = C'(\hat{e}_j) = b_j = C'(e_j^*), \tag{39}$$

so $\hat{e}_i = 0 < 1 = \hat{e}_j = e_j^*$ and the weak inequality holds. ■

Proof of Corollary 3: Let q be additively separable, $\alpha_i = \alpha$, and agents identical, so $c_i = c$. By Proposition 2, the optimal budget balancing bonus satisfies

$$\tilde{b}_i = 1 - \frac{\frac{c_i}{\alpha_i}(n-1)}{\sum_i \frac{c_i}{\alpha_i}} = 1 - \frac{n-1}{n} = \frac{1}{n}. \tag{40}$$

Let the salaries satisfy the participation constraint.

From Proposition 3, the equilibrium of the private contracting game satisfies budget balance, so $\sum \hat{b}_i = 1$. With identical agents, $\hat{b}_i = b$, so $\hat{b}_i = \frac{1}{n}$. And finally, in the principal's preferred equilibrium of the private contracting game, he will select salaries such that (PC_i') binds. Therefore, the optimal bonus is given by $b = \frac{1}{n}$, and the optimal salaries satisfy the participation constraint. ■

Proof of Proposition 4: Under full complementarity, the production function is $q(e) = \prod \beta_i e_i$, so its first derivative is

$$q_i(e) = \beta_i \prod_{j \neq i} \beta_j e_j \, . \tag{41}$$

Total surplus is

$$q(e) - \sum_{i=1}^{3} C_i(e_i). \tag{42}$$

With quadratic costs and $c_i = 1$, the FOC is

$$\beta e_j e_k = e_i \text{ for } i \neq j, k. \tag{43}$$

Under identical agents, $e_i = e$, so $e^* = \frac{1}{\beta} > 0$.

Let E_i be the externality on agent i when all other agents deviate from their equilibrium effort level, so

$$E_i = q(e) - q(e_i, \hat{e}_{-i}) = \beta_i e_i \left(\prod_{j \neq i} \beta_j e_j - \prod_{j \neq i} \beta_j \hat{e}_j \right) . \tag{44}$$

Differentiating with respect to each agent's effort gives

$$\frac{\partial E_i}{\partial e_i} = \beta \left(e_j e_k - \hat{e}_j \hat{e}_k \right) , \quad \frac{\partial E_i}{\partial e_j} = \beta e_i e_k \, , \quad \frac{\partial E_i}{\partial e_k} = \beta e_i e_j \, . \tag{45}$$

Differentiating with respect to bonus b gives

$$\frac{\partial E_i}{\partial b_i} = \beta_i e_i \left(\beta_j e_j \beta_k \frac{\partial e_k}{\partial b_i} + \beta_k e_k \beta_j \frac{\partial e_j}{\partial b_i} \right) . \tag{46}$$

The principal's optimization problem is to maximize

$$V = q(e) - \sum_{i=1}^{n} \Big(C_i(e_i) + b_i E_i \Big) . \tag{47}$$

By the chain rule,

$$\frac{\partial V}{\partial e_i} \frac{\partial e_i}{\partial b_i} + \frac{\partial V}{\partial e_j} \frac{\partial e_j}{\partial b_i} + \frac{\partial V}{\partial e_k} \frac{\partial e_k}{\partial b_i} + \frac{\partial V}{\partial b_i} = 0. \tag{48}$$

Identical agents have the same quadratic cost $C_i(e_i) = c_i e_i$. Normalize $c_i = 1$. Therefore the incentive constraint becomes

$$e_i = b_i \beta_i \prod_{j \neq i} \beta_j e_j \,. \tag{IC}$$

Let $\beta = \prod_i \beta_i > 0$. Solving for the effort levels in closed form gives

$$e_i = \beta^{-1} \left(\prod_{j \neq i}^{n} b_j \right)^{-1/2} . \tag{49}$$

Differentiating with respect to the bonus choices gives

$$\frac{\partial e_i}{\partial b_i} = 0 \,, \quad \frac{\partial e_i}{\partial b_j} = -\frac{b_k}{2\beta} \left(\prod_{j \neq i} b_j \right)^{-3/2} < 0 \,, \quad \frac{\partial e_i}{\partial b_k} = -\frac{b_j}{2\beta} \left(\prod_{j \neq i} b_j \right)^{-3/2} < 0 \tag{50}$$

As expected, there's no effect from increasing in individual's bonus but a negative effect from increasing another individual's bonus. Observe that in equilibrium $b = \hat{b}$, $E_i = 0$, and $\frac{\partial E_i}{\partial e_i} = 0$. Therefore, (48) simplifies to

$$\frac{\partial e_j}{\partial b_i} \left\{ q_j(e) - e_j - \left[b_i \frac{\partial E_i}{\partial e_j} + b_k \frac{\partial E_k}{\partial e_k} \right] \right\} + \frac{\partial e_k}{\partial b_i} \left\{ q_k(e) - e_k - \left[b_i \frac{\partial E_i}{\partial e_k} + b_j \frac{\partial E_j}{\partial e_k} \right] \right\} . \tag{51}$$

Because agents are identical, I can inspect the symmetric equilibrium where $b_i = b$ and therefore $e_i = e$. Substituting into (41) gives $q_i(e) = \beta e^2$ and each term in brackets in (51) becomes $b\beta e^2$. Therefore the first order condition from the principal's problem (51) reduces to

$$2(\beta e^2 - e - 2b\beta e^2)(-\frac{1}{2b^2\sqrt{\beta}}) = 0. \tag{52}$$

Rearranging and solving this generates $e = \Big(\beta(1-2b)\Big)^{-1}$. Under identical agents the incentive constraint is $e = \frac{1}{b\sqrt{\beta}}$. Combining these two gives the optimal bonus

$$\hat{b} = \frac{\beta}{\sqrt{\beta} + 2\beta}. \tag{53}$$

Observe that if $\beta = 1$, then $\hat{b} = \frac{1}{3}$ and $\sum \hat{b}_i = 1$. The optimal effort is

$$\hat{e} = \frac{\sqrt{\beta} + 2\beta}{\beta\sqrt{\beta}} = \left(1 + \frac{2\beta}{\sqrt{\beta}} \right) \frac{1}{\beta} > \frac{1}{\beta} = e^*. \tag{54}$$

Now

$$\frac{\partial b}{\partial \beta} = \frac{\sqrt{\beta} - \dfrac{\beta}{2\sqrt{\beta}}}{\left(\sqrt{\beta} + 2\beta\right)^2} > 0. \tag{55}$$

And $\frac{\partial \hat{e}}{\partial \beta} < 0$ since $2\beta + 2\beta\sqrt{\beta} > 0$.

■

7 References

1. Baldenius, T.; S. Dutta; and S. Reichelstein. "Cost Allocation for Capital Budgeting Decisions." *The Accounting Review* (2007): 837-867.

2. Demski, J. S. 1981. Cost allocation games. *Joint Cost Allocations*, edited by S. Moriarity, 142-73. Center for Economic and Management Research, University of Oklahoma, Norman.

3. Eswaran, Mukesh, and Ashok Y. Kotwal. 1984. "The Moral Hazard of Budget-Breaking." Rand Journal of Economics, 15(4): 578-81.

4. Holmström, B. "Moral Hazard in Teams." *Bell Journal of Economics* 13 (1982): 324-340.

5. Holmström, B., and P. Milgrom. "Regulating Trade Among Agents." *Journal of Institutional and Theoretical Economics* (1990): 85-105.

6. Huddart, S., and P. J. Liang. "Profit sharing and monitoring in partnerships." *Journal of Accounting and Economics* 40 (2005): 153-187.

7. Itoh, H. "Coalitions, incentives, and risk sharing." *Journal of Economic Theory* 60 (1993): 410-427.

8. Kofman, F., and J. Lawarree. "Collusion in hierarchical agency." *Econometrica: Journal of the Econometric Society* (1993): 629-656.

9. Lazear EP, and Rosen S. "Rank-Order Tournaments as Optimum Labor Contracts." *Journal of Political Economy* 89(1981): 841-864.

10. Legros, P., and H. Matsushima. "Efficiency in partnerships." *Journal of Economic Theory* 55 (1991): 296-322.

11. Legros, P., and S. Matthews. "Efficient and nearly-efficient partnerships." *The Review of Economic Studies* 60 (1993): 599-611.

12. Macho-Stadler, I., and J. D. Perez-Castrillo. "Moral hazard with several agents: The gains from cooperation." *International Journal of Industrial Organization* 11 (1993): 73-100.

13. McAfee, R. P., and M. Schwartz. "Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity, and Uniformity." *The American Economic Review* 84 (1994): 210230.

14. Miller, N. "Efficiency in partnerships with joint monitoring." *Journal of Economic Theory* 77 (1997): 285-299.

15. Rajan, M. "Cost allocation in multiagent settings." *The Accounting Review* (1992):

527-545.

16. Ramakrishnan, R. T., and A. V. Thakor. "Cooperation versus competition in agency." *Journal of Law, Economics, and Organization* (1991): 248-283.

17. Rasmusen, E. "Moral hazard in risk-averse teams." *The RAND Journal of Economics* (1987): 428-435.

18. Ray, K., and M. Goldmanis. "Efficient Cost Allocation." *Management Science* (2012): 1341-1356.

19. Segal, I. "Contracting with Externalities." *The Quarterly Journal of Economics* 114 (1999): 337388.

20. Segal, I., and M. D. Whinston. "Robust Predictions for Bilateral Contracting with Externalities." *Econometrica* 71 (2003): 757791.

21. Varian, H. "Monitoring agents with other agents." *Journal of Institutional and Theoretical Economics* (1990): 153-174.